TRANSFORMER

Introduction

The main advantage of alternating currents over direct currents is that the alternating currents can be easily transferable from low voltage to high or from high voltage to low. Alternating voltages can be raised or lowered in the different stages of electrical network as generation, transmission, distribution and utilization. This is possible with a static device is called transformer. The transformer works on the principle of mutual induction. It transfers an electric energy from one circuit to another (with the desired change in voltage and current, without any change in the frequency) when there is no electrical connection between the two circuits.

1. Principle of Working

The principle of mutual induction states that when two coils are inductively coupled and if current in one coil is changed uniformly then an e.m.f. gets induced in the other coil. This e.m.f. can drive a current, when a closed path is provided to it.

The transformer works on the same principle. In its elementary form, it consists of two inductive coils (have high mutual inductance) which are electrically separated but linked through a common magnetic circuit. The basic transformer is shown in the Figure (1).

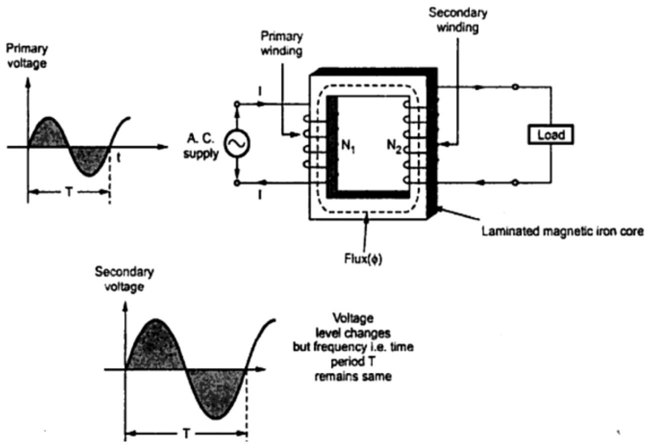


Figure (1): Basic transformer

One of the two coils is connected to a source of alternating voltage. This coil is called primary winding (P). The other winding is connected to load. This winding is called secondary winding (S). The primary winding has N_1 number of turns while the secondary winding has N_2 number of turns. Symbolically the transformer is indicated as shown in the Figure (2).

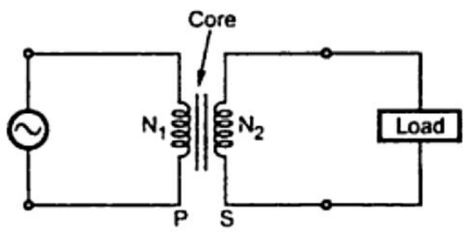


Figure (2): Symbolic representation

If primary winding is connected to a source of alternating voltage, an alternating flux is set up in the laminated core, most of which is linked with the secondary winding in which it produces mutually-induced e.m.f. (according to *Faraday's Laws of Electromagnetic Induction* $e = MdI/dt$). If the secondary winding is closed, a current flows in it and so electric energy is transferred (entirely magnetically) from the primary winding to secondary winding.

Can D.C. Supply be used for Transformers ?

The d.c. supply can not be used for the transformers.

The transformer works on the principle of mutual induction, for which current in one coil must change uniformly. If d.c. supply is given, the current will not change due to constant supply and transformer will not work.
Practically winding resistance is very small. For d.c, the inductive reactance X_L is zero as d.c has no frequency. So total impedance of winding is very low for d.c. Thus winding will draw very high current if d.c. supply is given to it. This may cause the burning of windings due to extra heat generated and may cause permanent damage to the transformer.
There can be saturation of the core due to which transformer draws very large current from the supply when connected to d.c. Thus d.c. supply should not be connected to the transformers.

2. Construction

There are two basic parts of a transformer

i) Magnetic core   ii) Winding or coils.

The core of the transformer is either square or rectangular in size. It is further divided into two parts. The vertical portion on which coils are wound is called limb while the top and bottom horizontal portion is called yoke of the core. These parts are shown in the Figure (3-a).

Core is made up of laminations to reduce eddy current losses. Generally high grade silicon steel laminations [0.3 to 0.5 mm thick] are used to reduce hysteresis loss. These laminations are insulated from each other by using insulation like varnish. Laminations are overlapped so that to avoid the air gap at the joints. For this generally **L** shaped or **T** shaped laminations are used which are shown in the Figure (3-b).

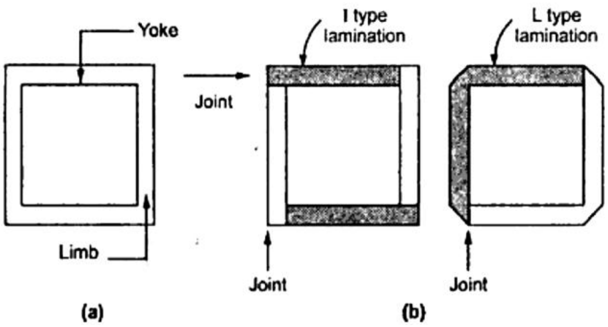


Figure (3): Construction of transformer

The cross-section of the limb depends on the type of coil to be used either circular or rectangular. The different cross-sections of limbs, practically used are shown in the Figure (4).

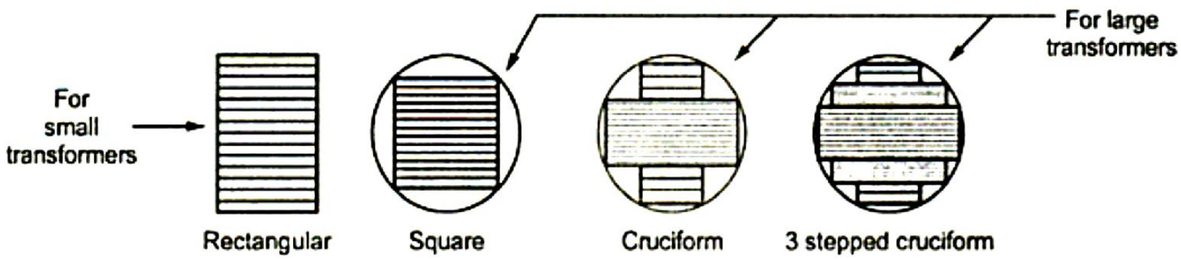


Figure (4): Different cross section

3. Types of Windings

The coils used are wound on the limbs and are insulated from each other. In the basic transformer shown in the Figure (1), the two windings wound are shown on two different limbs i.e. primary on one limb while secondary on other limb. But due to this leakage flux increases which affects the transformer performance badly. Similarly it is necessary that the windings should be very closes to each other to have high mutual inductance. To achieve this, the two windings are split into number of coils and are wound adjacent to each other on the, same limb. A very common arrangement is cylindrical concentric coils as shown in the Figure (5).

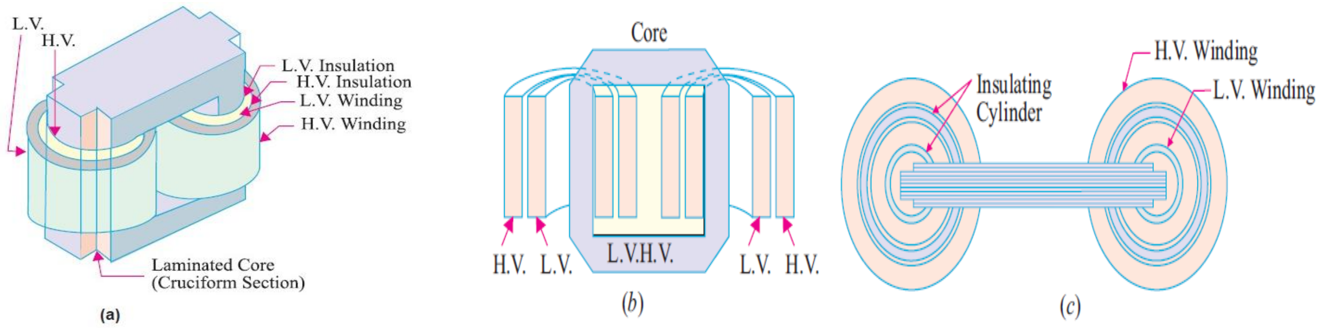


Figure (5): Cylindrical concentric coils

Such cylindrical coils are used in the core type transformer. These coils are mechanically strong. These are wound in the helical layers. The different layers are insulated from each other by paper, cloth or mica. The low voltage winding is placed near the core from ease of insulating it from the core. The high voltage is placed after it.

The other type of coils which is very commonly used for the shell type of transformer is sandwich coils. Each high voltage portion lies between the two low voltage portion sandwiching the high voltage portion. Such subdivision of windings into small portions reduces the leakage flux. Higher the degree of subdivision, smaller is the reactance. The sandwich coil is shown in the Figure (6). The top and bottom coils are low voltage coils. All the portions are insulated from each other by paper.

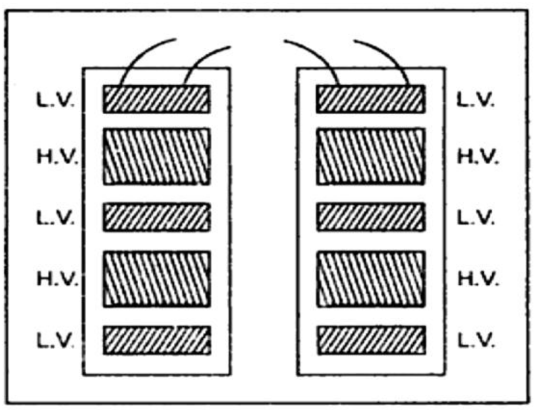


Figure (6): Sandwich coils

4. Types of Transformers

The classification of the transformers is based on the relative arrangement or disposition of the core and the windings. There are three main types of the transformers which are:

i) Core type ii) Shell type and iii) Berry type

i) Core Type Transformer

It has a single magnetic circuit. The core is rectangular having two limbs. The winding encircles the core [Figure (7)]. The coils used are of cylindrical type. Core is made up of large number of thin laminations. The coils can be easily removed by removing the laminations of the top yoke, for maintenance.

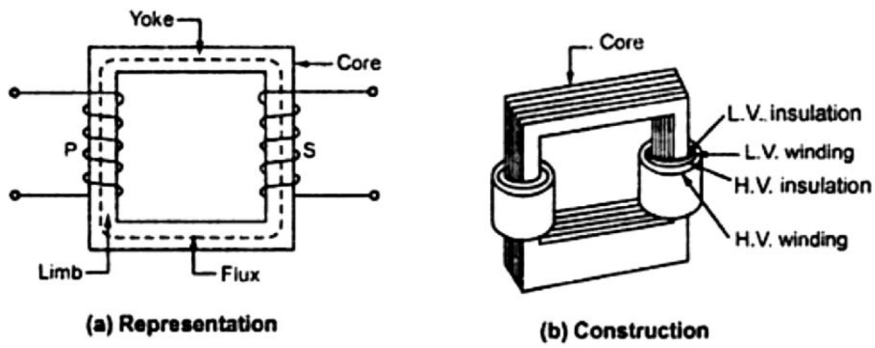


Figure (7): Core type transformer

ii) Shell Type Transformer

It has a double magnetic circuit. The core has three limbs as shown in Figure (8). Both the windings are placed on the central limb. The core encircles most part of the windings. The coils used are generally multilayer disc type or sandwich coils.

Generally for very high voltage transformers, the shell type construction is preferred. For removing any winding for maintenance, large number of laminations are required to be removed.

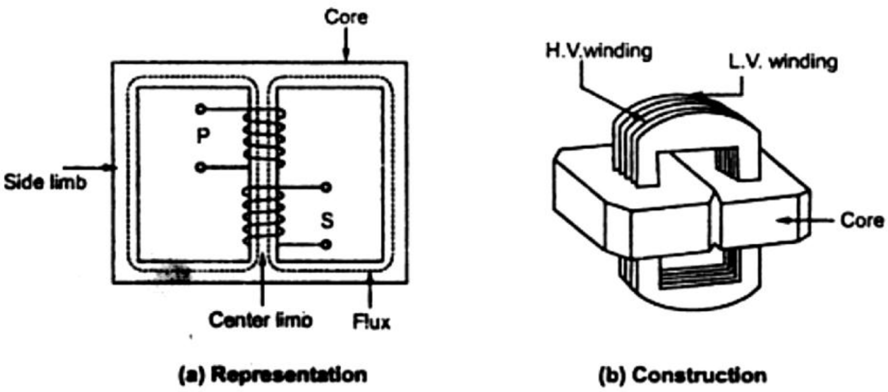


Figure (8): Shell type transformer

iii) Berry Type Transformer

This has distributed magnetic circuit The number of independent magnetic circuits are more than 2. Its core construction is like spokes of a wheel. Otherwise it is symmetrical to that of shell type as shown as in the Figure (9).

The transformers are generally kept in tightly fitted sheet metal tanks. The tanks are constructed of specified high quality steel plate. The tanks are filled with the special insulating oil. The entire transformer assembly is immersed in the oil. The oil serves two functions:

i)Keeps the coils cool and **ii)**Provides the transformers an additional insulation.

The oil should be absolutely free from alkalies, sulphur and specially from moisture. Presence of very small moisture lowers the dielectric strength of oil, affecting its performance badly.

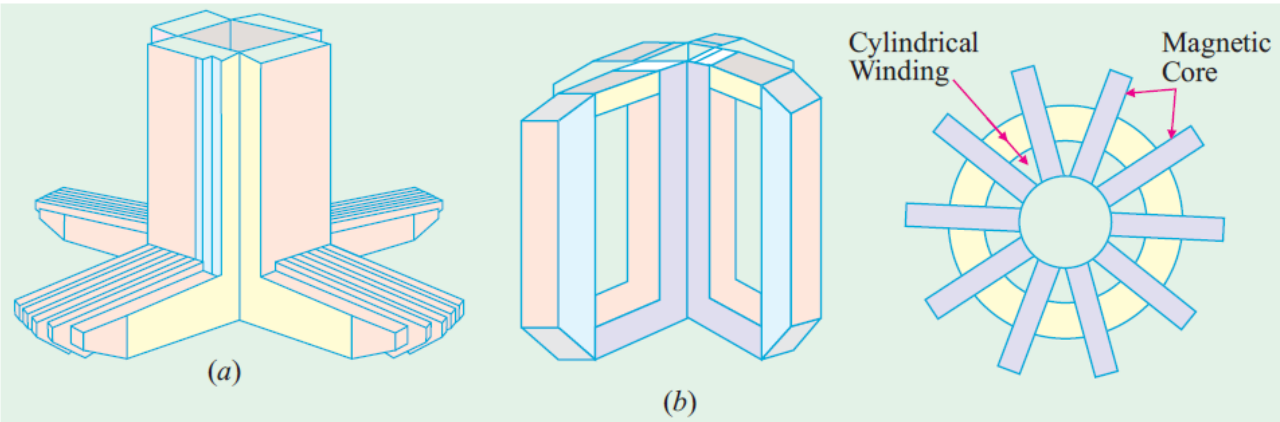


Figure (9): Berry type transformer

Table (1): Comparison between core and shell type transformers

	Core Type	Shell Type
1	The winding encircles the core.	The core encircles most part of the windings.
2	It has single magnetic circuit.	It has double magnetic circuits.
3	The core has two limbs.	The core has three limbs.
4	The cylindrical coils are used.	The multilayer disc or sandwich type coils are used.
5	The coils can be easily removed from maintenance point of view.	The coils cannot be removed easily.
6	Preferred for low voltage transformers.	Preferred for high voltage transformers.

5. E.M.F Equation of Transformer

When the primary winding is excited by an alternating voltage V_t, it circulates alternating current, producing an alternating flux Φ. The primary winding has N_1 number of turns. The alternating flux Φ linking with the primary winding itself induces an e.m.f. in it denoted as E_1. The flux links with secondary winding through the common magnetic core. It produces induced e.m.f. E_2 in the secondary winding. This is mutually induced e.m.f. Let us derive the equations for E_1 and E_2.

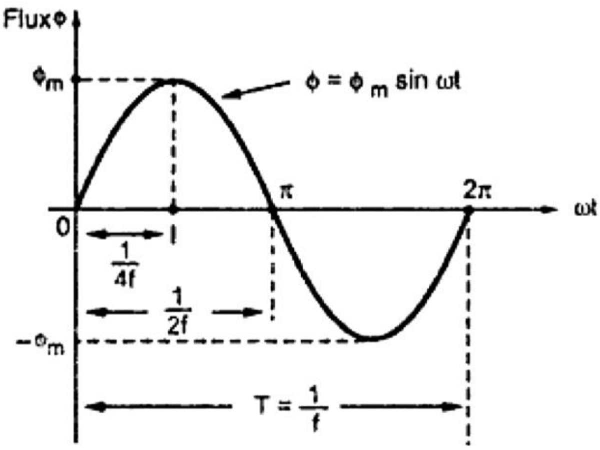


Figure (10): Sinusoidal flux

The primary winding is excited by purely sinusoidal alternating voltage. Hence the flux produced is also sinusoidal in nature having maximum value of Φ_m as shown in the Figure (10).

The various quantities which affect the magnitude of the induced e.m.f. are :

Φ = Flux

Φ_m = Maximum value of flux

N_1 = Number of primary winding turns

N_2 = Number of secondary winding turns

f = Frequency of the supply voltage

E_1 = R.M.S. value of the primary induced e.m.f.

E_2 = R.M.S. value of the secondary induced e.m.f.

From *Faraday's law of electromagnetic induction* the average e.m.f. induced in each turn is proportional to the average rate of change of flux.

∴ Average e.m.f. per turn = Average rate of change of flux

∴ Average e.m.f. per turn = $\frac{d\Phi}{dt}$

Now $\frac{d\Phi}{dt} = \frac{\text{change in flux}}{\text{Time required for change in flux}}$

Consider the $(1/4)^{th}$ cycle of the flax as shown in the Figure (10). Complete cycle gets completed in (1/f) seconds. In $(1/4)^{th}$ time period, the change in flux is from 0 to Φ_m .

$$\frac{d\Phi}{dt} = \frac{\Phi_m - 0}{\left(\frac{1}{4f}\right)}$$ as (dt) for $(1/4)^{th}$ time period is (l/4f) seconds

$= 4f\ \Phi_m$ Wb/sec.

∴ Average e.m.f./turn = $4f\ \Phi_m$ volts

As Φ is sinusoidal, the induced e.m.f. in each turn of both the windings is also sinusoidal in nature. For sinusoidal quantity,

Form Factor (F.F.) = $\frac{\text{R.M.S.value}}{\text{Average value}} = 1.11$

∴ R.M.S. value = 1.11 × Average value

∴ R.M.S. value of induced e.m.f. /turn= $1.11 \times 4f\ \Phi_m = 4.44f\ \Phi_m$

∴ R.M.S. value of induced e.m.f. for whole primary windings (E_1)= $N_1 \times 4.44f\ \Phi_m$ volts

∴ R.M.S. value of induced e.m.f. for whole secondary windings (E_2)= $N_2 \times 4.44f\ \Phi_m$ volts

The expressions of E_1 & E_2 are called e.m.f. equations of transformer:

$$E_1 = 4.44f\ \Phi_m\ N_1 \text{ volts} \qquad \ldots(1)$$

$$E_2 = 4.44f\ \Phi_m\ N_2 \text{ volts} \qquad \ldots(2)$$

It is seen from (1) and (2) that $[(E_1/N_1) = (E_2/N_2) = 4.44\ f\ \Phi_m]$. It means that e.m.f./turn is the same in both the primary and secondary windings.

6. Ratio of Transformer

(i) Voltage ratio:

From the e.m.f. equation of transformer, $E_1 = 4.44f\ \Phi_m\ N_1$ & $E_2 = 4.44f\ \Phi_m\ N_2$.
Taking the ratio of the two equations we get,

$$\boxed{\frac{E_2}{E_1} = \frac{N_2}{N_1} = K}$$

This ratio secondary induced e.m.f. to primary induced e.m.f. is known as *voltage transformation ratio* denoted as K, while $\left(\frac{N_1}{N_2}\right)$ known as turn ratio.

Thus, $\boxed{E_2 = KE_1 \qquad \text{where } K = \frac{N_2}{N_1}}$

1) If $N_2 > N_1$ (i.e. $K > 1$), we get $E_2 > E_1$ then the transformer is called *'step-up transformer'*.
2) If $N_2 < N_1$ (i.e. $K < 1$), we get $E_2 < E_1$ then the transformer is called *'step-down transformer'*.
3) If $N_2 = N_1$ (i.e. $K = 1$), we get $E_2 = E_1$ then the transformer is called *'isolation transformer'* or *1:1 transformer.*

(ii) Current Ratio

For an ideal transformer there are no losses. Hence the product of primary voltage V_1 and primary current I_1, is same as the product of secondary voltage V_2 and the secondary current I_2.

So $V_1 I_1 =$ input VA and $V_2 I_2 =$ output VA

For an ideal transformer, $V_1 I_1 = V_2 I_2$

$$\boxed{\frac{V_2}{V_1} = \frac{I_1}{I_2} = K}$$

7. Ideal Transformer

A transformer is said to be ideal if it satisfies following properties :

i) It has no losses.

ii)Its windings have zero resistance.

iii) Leakage flux is zero i.e. 100 % flux produced by primary links with the secondary.

iv)Permeability of core is so high that negligible current is required to establish the flux in it.

Note: For an ideal transformer, the primary applied voltage V_t is same as the primary induced e.m.f. E_1 as there are no voltage drops. Similarly the secondary induced e.m.f. E_2 is also same as the terminal voltage V_2 across the load. Hence for an ideal transformer we can write,

$$\boxed{\frac{E_2}{E_1} = \frac{V_2}{V_1} = K}$$

No transformer is ideal in practice but the value of E_1 is almost equal to V_1 for properly designed transformer.

Example 1: The maximum flux density in the core of a 250/3000-volts, 50-Hz single-phase transformer is 1.2 Wb/m^2. If the e.m.f. per turn is 8 volt, determine, **(i)** primary and secondary turns **(ii)** area of the core.

Solution:

Note: 250/3000-volts, represent (primary voltage/secondary voltage).

(i) $E_1 = N_1 \times$ e.m.f. induced/turn

$N_1 = 250/8 = \mathbf{32}$; $N_2 = 3000/8 = \mathbf{376}$

(ii) We may use $E_2 = -\,4.44\ f\ N_2\ B_m\ A$

$\therefore\ 3000 = 4.44 \times 50 \times 375 \times 1.2 \times A$

$\therefore\ \mathbf{A = 0.03m^2.}$

Example 2: The core of a 100-kVA, 11000/550 V, 50-Hz, 1-ph, core type transformer has a cross-section of 20 cm × 20 cm. Find **(i)** the number of H.V. and L.V. turns per phase and **(ii)** the e.m.f. per turn if the maximum core density is not to exceed 1.3 Tesla. Assume a stacking factor of 0.9.

Solution:

(i) $B_m = 1.3$ T,

Note: when a stacking factor is given, area of core must be multiply by this factor.

$A = (0.2 \times 0.2) \times 0.9 = 0.036\ m^2$

$\therefore\ 11{,}000 = 4.44 \times 50 \times N_1 \times 1.3 \times 0.036 \quad \Rightarrow \quad N_1 = 1060$

$550 = 4.44 \times 50 \times N_2 \times 1.3 \times 0.036 \quad \Rightarrow \quad N_2 = 54$

or, $N_2 = KN_1 = (550/11{,}000) \times 1060 = 54$

(ii) e.m.f./turn = 11,000/1060 = 10.4 V or 550/54 = 10.2 V

Example 3: A single-phase transformer has 400 primary and 1000 secondary turns. The net cross-sectional area of the core is 60 cm^2. If the primary winding be connected to a 50-Hz supply at 520 V, calculate **(i)** the peak value of flux density in the core **(ii)** the voltage induced in the secondary winding.

Solution:

$K = N_2/N_1 = 1000/400 = 2.5$

(i) $E_2/E_1 = K$, $\quad \therefore\ E_2 = KE_1 = 2.5 \times 520 = \mathbf{1300\ V}$

(ii) $E_1 = 4.44\ f\ N_1\ B_m A$ or $520 = 4.44 \times 50 \times 400 \times B_m \times (60 \times 10^{-4})$

$\therefore\ B_m = \mathbf{0.976\ Wb/m^2}$

Example 4: A 25-kVA transformer has 500 turns on the primary and 50 turns on the secondary winding. The primary is connected to 3000-V, 50-Hz supply. Find the full-load primary and secondary currents, the secondary e.m.f. and the maximum flux in the core. Neglect leakage drops and no-load primary current.

Solution:

$K = N_2/N_1 = 50/500 = 1/10$

F.L. $I_1 = 25{,}000/3000 =$ **8.33 A &** F.L. $I_2 = I_1/K = 10 \times 8.33 =$ **83.3 A**

e.m.f. per turn on primary side = 3000/500 = 6 V

∴ secondary e.m.f. = 6 × 50 = **300 V** (or $E_2 = KE_1 = 3000 \times 1/10 = 300$ V)

$E_1 = 4.44\, f\, N_1\Phi_m$;

$3000 = 4.44 \times 50 \times 500 \times \Phi_m$

∴ Φ_m **= 27 mWb**

Example 5: A single phase transformer has 500 turns in the primary and 1200 turns in the secondary. The cross-sectional area of the core is 80 cm^2. If the primary winding is connected to a 50 Hz supply at 500 V, calculate **(i)** Peak flux-density, and **(ii)** Voltage induced in the secondary.

Solution:

From the e.m.f. equation for transformer,

$500 = 4.44 \times 50 \times \Phi_m \times 500 \Longrightarrow \Phi_m = (1/222)$ Wb

(i) Peak flux density, $B_m = \Phi_m / (80 \times 10^{-4}) = 0.563$ Wb/m^2

(ii) Voltage induced in secondary is obtained from transformation ratio or turns ratio

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \Longrightarrow V_2 = 500 \times (1200/500) = 1200 \text{ volts}$$

Example 6: A 25 kVA, single-phase transformer has 250 turns on the primary and 40 turns on the secondary winding. The primary is connected to 1500-volt, 50 Hz mains. Calculate **(i)** Primary and Secondary currents on full-load, **(ii)** Secondary e.m.f., **(iii)** maximum flux in the core.

Solution:

(i) If V_2 = Secondary voltage rating, = secondary e.m.f.

$$\frac{V_2}{1500} = \frac{40}{250} \Longrightarrow \therefore V_2 = 240 \text{ volts}$$

(ii) Primary current = 25000/1500 = 16.67 A & Secondary current = 25000/240 = 104.2 A

(iii) If Φ_m is the maximum core-flux in Wb,

$1500 = 4.44 \times 50 \times \Phi_m \times 250$

∴ $\Phi_m = 0.027$ Wb = 27 mWb

Example 7: A single-phase, 50 Hz, core-type transformer has square cores of 20 cm side. Permissible maximum flux-density is 1 Wb/m^2. Calculate the number of turns per Limb on the High and Low-voltage sides for a 3000/220 V ratio.

Solution:

$220 = 4.44 \times 50 \times [(20 \times 20 \times 10^{-4}) \times 1] \times N_2$

$N_2 = 220/8.88 = 24.77$, Select $N_2 = 26$

$N_1/N_2 = V_1/V_2$

$\therefore N_1 = N_2 \times (V_1/V_2) = 26 \times 3000/220 = 356$, *selecting the nearest even integer.*

Number of H.V. turns on each Limb = 178 & Number of L.V. turns on each Limb = 14

8. Ideal Transformer on No Load

An ideal transformer is one which has no losses *i.e.* its windings have no ohmic resistance, there is no magnetic leakage and hence which has no I^2R and core losses. In other words, it consists of two purely inductive coils wound on a loss-free core.

Consider an ideal transformer [Figure (11)] who's secondary is open and whose primary is connected to sinusoidal alternating voltage V_1. This potential difference causes an alternating current to flow in the primary. Since the primary coil is purely inductive and there is no output (secondary being open) so the primary draws the magnetising current I_μ only. The function of this current is merely to magnetise the core, it is small in magnitude and lags V_1 by 90°. This alternating current I_μ produces an alternating flux φ which is, at all times, proportional to the current (assuming permeability of the magnetic circuit to be constant) and, hence, is in phase with it. This changing flux is linked both with the primary and the secondary windings. Therefore, it produces self-induced e.m.f. in the primary.

According to *Lenz's law*, the induced e.m.f opposes the cause producing it which is supply voltageV_1. Hence E_1 (*self-induced* e.m.f. or) is in anti phase with V_1 but equal in magnitude. E_1 is also known as counter e.m.f. or back e.m.f. of the primary. Similarly, there is produced in the secondary an induced e.m.f. E_2 which is known as *mutually* induced e.m.f. This e.m.f. is anti phase with V_1 and its magnitude is proportional to the rate of change of flux and the number of secondary turns.

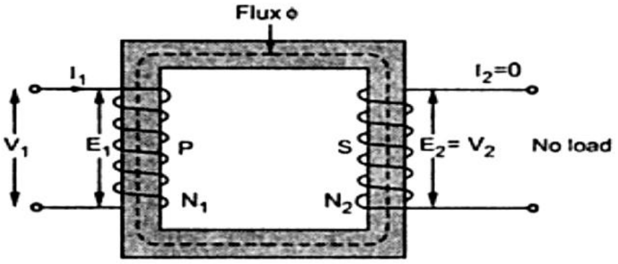


Figure (11): Ideal transformer on no load

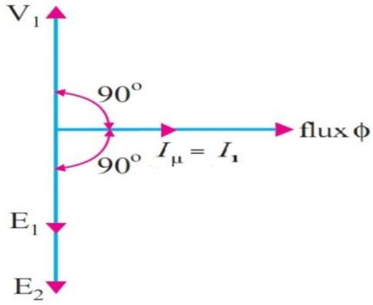


Figure (12): phasor diagram for ideal transformer on no load.

Note:

- The power input to the transformer is [$V_1I_1\cos(90)$= zero]. This is because on N.L. output power is zero and for ideal transformer there are no losses hence input power is also zero.
- Ideal transformer power factor (p.f.) on no load is zero lagging.

9. Practical Transformer with Losses but no Magnetic Leakage

We will consider two cases ***(i)*** when such a transformer is on N.L. ***(ii)*** when it is loaded.

(i) Transformer on No-load

The primary input current under no-load conditions has to supply ***(i)*** iron losses in the core *i.e.* hysteresis loss and eddy current loss and ***(ii)*** a very small amount of copper loss in primary (there being no Cu loss in secondary as it is open). Hence, the no-load primary input current I_0 is not at 90° behind V_1 but lags it by an angle φ_0 < 90°. No-load input power,

$\boxed{W_0 = V_1 I_0 \cos\varphi_0}$ where ($\cos\varphi_0$) is primary power factor under no-load.

As seen from phasor diagram on No-load condition of an actual transformer [Figure (13)], primary current I_0 has two components:

(i) One in phase with V_1. This is known as ***active*** or ***working*** or ***iron*** loss component I_w because it mainly supplies the iron loss plus small quantity of primary Cu loss.

$$\boxed{I_w = I_0 \cos\varphi_0}$$

(ii) The other component is in quadrature with V_1 and is known as ***magnetizing*** component I_μ because its function is to sustain the alternating flux in the core. It is wattless.

$$\boxed{I_\mu = I_0 \sin\varphi_0}$$

Obviously, I_0 is the vector sum of I_w and I_μ, hence

$$\boxed{I_0 = \sqrt{(I_\mu^2 + I_\omega^2)}}$$

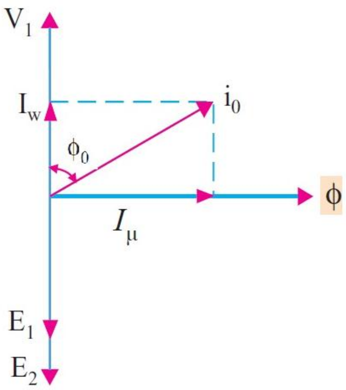


Figure (13)

The following points should be noted carefully:

1) The no-load primary current I_0 is very small as compared to the full-load primary current. It is about 1 per cent of the full-load current.

2) Owing to the fact that the permeability of the core varies with the instantaneous value of the exciting current, the wave of the exciting or magnetising current is not truly sinusoidal. As such it should not be represented by a vector because only sinusoidally varying quantities are represented by rotating vectors. But, in practice, it makes no appreciable difference.

3) As I_0 is very small, the no-load primary Cu loss is negligibly small which means ***that no-load primary input is practically equal to the iron loss in the transformer***.

4) As it is principally the core-loss which is responsible for shift in the current vector, angle φ_0 is known as ***hysteresis angle of advance***.

Note: The total power input on no load is denoted as W_0 and is given by,

$$\boxed{W_0 = V_1 I_0 \cos\varphi_0 = V_1 I_w}$$

Example 8: (a) A 2,200/200-V transformer draws a no-load primary current of 0.6 A and absorbs 400 watts. Find the magnetising and iron loss currents. **(b)** A 2,200/250-V transformer takes 0.5 A at a p.f. of 0.3 on open circuit. Find magnetizing and working components of no-load primary current.

Solution:

(a) Iron-loss current $= \dfrac{\text{no-load input in watts}}{\text{primary voltage}} = \dfrac{400}{2200} = \mathbf{0.182\ A}$

$I_0^2 = I_w^2 + I_\mu^2$

Magnetising component $I_\mu = \sqrt{(0.6^2 - 0.182^2)} = \mathbf{0.572\ A}$

(b) $I_0 = 0.5$ A, $\cos\varphi_0 = 0.3$

$\therefore\ I_w = I_0 \cos\varphi_0 = 0.5 \times 0.3 = 0.15$ A

$I_\mu = \sqrt{0.5^2 - 0.15^2} = 0.476$ A

Tutorial Problems (1)

[1] The number of turns on the primary and secondary windings of a 1−φ transformer are 350 and 35 respectively. If the primary is connected to a 2.2 kV, 50-Hz supply, determine the secondary voltage on no-load. **[220 V]**

[2] A 3000/200-V, 50-Hz, 1-phase transformer is built on a core having an effective cross-sectional area of 150 cm^2 and has 80 turns in the low-voltage winding. Calculate
(*a*) the value of the maximum flux density in the core
(*b*) the number of turns in the high-voltage winding. **[(*a*) 0.75 Wb/m^2 (*b*) 1200]**

[3] A 3,300/230-V, 50-Hz, 1-phase transformer is to be worked at a maximum flux density of 1.2 Wb/m^2 in the core. The effective cross-sectional area of the transformer core is 150 cm2. Calculate suitable values of primary and secondary turns. **[826; 58]**

[4] A 40-kVA, 3,300/240-V, 50 Hz, 1-phase transformer has 660 turns on the primary. Determine
(*a*) the number of turns on the secondary
(*b*) the maximum value of flux in the core
(*c*) the approximate value of primary and secondary full-load currents.
Internal drops in the windings are to be ignored.
[(*a*) 48 (*b*) 22.5 mWb (*c*) 12.1 A; 166.7 A]

[5] A double-wound, 1-phase transformer is required to step down from 1900 V to 240 V, 50-Hz. It is to have 1.5 V per turn. Calculate the required number of turns on the primary and secondary windings respectively. The peak value of flux density is required to be not more than 1.2 Wb/m^2. Calculate the required cross-sectional area of the steel core. If the output is 10 kVA, calculate the secondary current. [**1,267; 160; 56.4 cm^2; 41.67 A**]

[6] The no-load voltage ratio in a 1-phase, 50-Hz, core-type transformer is 1,200/440. Find the number of turns in each winding if the maximum flux is to be 0.075 Wb. [**74 and 28 turns**]

[7] A 1-phase transformer has 500 primary and 1200 secondary turns. The net cross-sectional area of the core is 75 cm^2. If the primary winding be connected to a 400-V, 50 Hz supply, calculate. (*i*) the peak value of flux density in the core and (*ii*) voltage induced in the secondary winding. [**0.48 Wb/m^2; 960 V**]

[8] A 10-kVA, 1-phase transformer has a turn ratio of 300/23. The primary is connected to a 1500-V, 60 Hz supply. Find the secondary volts on open-circuit and the approximate values of the currents in the two windings on full-load. Find also the maximum value of the flux. **[115 V; 6.67 A; 87 A; 18.76 mWb]**

[9] A 100-kVA, 3300/400-V, 50 Hz, 1 phase transformer has 110 turns on the secondary. Calculate the approximate values of the primary and secondary full-load currents, the maximum value of flux in the core and the number of primary turns.
How does the core flux vary with load ? **[30.3 A; 250 A; 16.4 mWb; 908]**

[10] The no-load current of a transformer is 5.0 A at 0.3 power factor when supplied at 230-V, 50-Hz. The number of turns on the primary winding is 200. Calculate (*i*) the maximum value of flux in the core (*ii*) the core loss (*iii*) the magnetising current. **[5.18 mWb; 345 W; 4.77 A]**

[11] The no-load current of a transformer is 15 at a power factor of 0.2 when connected to a 460-V, 50-Hz supply. If the primary winding has 550 turns, calculate
(*a*) the magnetising component of no-load current
(*b*) the iron loss
(*c*) the maximum value of the flux in the core. **[(*a*) 14.7 A (*b*) 1,380 W (*c*) 3.77 mWb]**

[12] The no-load current of a transformer is 4.0 A at 0.25 p.f. when supplied at 250-V, 50 Hz. The number of turns on the primary winding is 200. Calculate
(*i*) the r.m.s. value of the flux in the core (assume sinusoidal flux)
(*ii*) the core loss
(*iii*) the magnetising current. **[(*i*) 3.96 mWb (*ii*) 250 W (*iii*) 3.87 A]**

[13] The following data apply to a single- phase transformer:
output : 100 kVA, secondary voltage; 400 V; Primary turns: 200; secondary turns: 40;
Neglecting the losses, calculate: (*i*) the primary applied voltage (*ii*) the normal primary and secondary currents (*iii*) the secondary current, when the load is 25 kW at 0.8 power factor.
[(*i*) 2000 V, (*ii*) 50 amp, & 250 amp. (*iii*) 78.125 amp]

(ii) Transformer on Load

When the transformer is loaded, the current I_2 flows through the secondary winding. The magnitude and phase of I_2 is determined by the load. If load is inductive, I_2 lags V_2. If load is capacitive, I_2 leads V_2 while for resistive load; I_2 is in phase with V_2.

There exists a secondary m.m.f. (N_2I_2) due to which secondary current sets up its own flux This flux opposes the main flux Φ which is produced in the core due to magnetising component of no load current Hence the m.m.f (N_2I_2) is called *demagnetising ampere-turns*. This is shown in the Figure (14-a).

The flux Φ_2 momentarily reduces the main flux Φ, due to which the primary induced e.m.f. E_l also reduces. Hence the vector difference $(\overrightarrow{V_1} - \overrightarrow{E_1})$ increases due to which primary draws more current from the supply. This additional current drawn by primary is due to the load hence called load component of primary current denoted as I_2' as shown in the Figure (14-b).

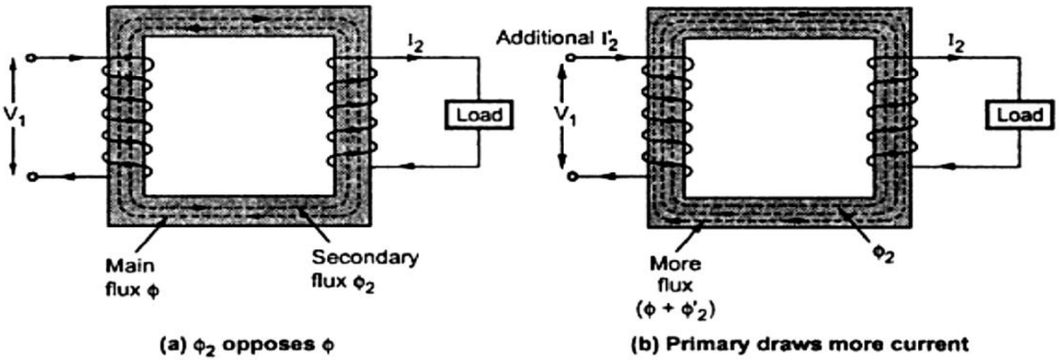


Figure (14): Transformer on load

This current I_2' is in antiphase with I_2. The current I_2 sets up its own flux Φ_2' which opposes the flux Φ_2 and helps the main flux Φ. This flux Φ_2' neutralises the flux Φ_2 produced by I_2. The m.m.f. i.e. ampere turns (N_1I_2') balances the ampere turns (N_2I_2). Hence the net flux in the core is again maintained at constant level.

Note: For any load condition, N.L. to F.L.the flux in the core is practically constant.

The load component current I_2' always neutralises the changes in the load. As practically flux in core is constant, the core loss is also constant for all the loads. Hence the transformer is called constant flux machine.

As the ampere turns are balanced we can write,

$N_2I_2 = N_1I_2'$

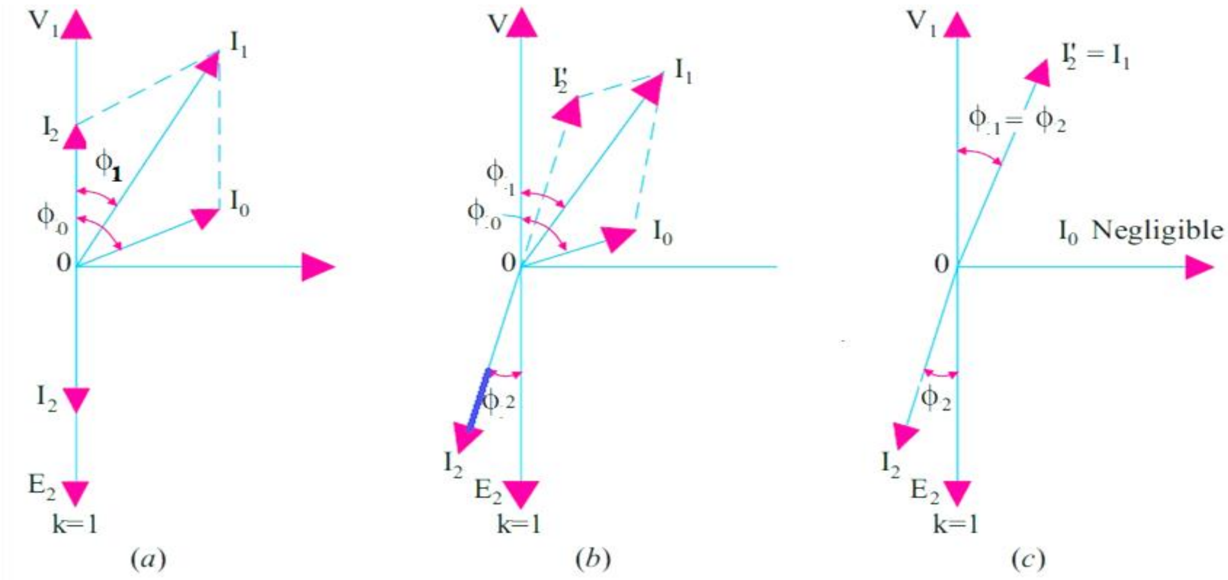
$$\boxed{I_2' = \frac{N_2}{N_1} I_2 = KI_2}$$

Thus when transformer is loaded, the primary current I_1 has two components :

1) The no load current I_0 which lags V_1 by angle φ_0. It has two components I_μ and I_w.
2) The load component I_2' which is in antiphase with I_2. And phase of I_2 is decided by the load.

Hence primary current I_1 is vector sum of I_0 and I_2'.

$$\therefore \quad \vec{I_1} = \vec{I_0} + \vec{I_2'}$$



Resistive **Inductive** **Inductive with I_o == 0**

Figure (15)

In Figure (15) are shown the vector diagrams for a load transformer when load is non-inductive and when it is inductive (a similar diagram could be drawn for capacitive load). Voltage transformation ratio of unity is assumed so that primary vectors are equal to the secondary vectors. With reference to Figure (15-a), I_2 is secondary current in phase with E_2 (strictly speaking it should be V_2). It causes primary current I_2' which is anti-phase with it and equal to it in magnitude ($K = 1$). Total primary current I_1 is the vector sum of I_0 and I_2' and lags behind V_1 by an angle φ_1.

In Figure (15-b) vectors are drawn for an inductive load. Here I_2 lags E_2 (actually V_2) by φ_2. Current I_2' is again antiphase with I_2 and equal to it in magnitude. As before, $I_1$is the vector sum of I_2' and I_0 and lags behind V_1 by φ_1.

It will be observed that φ_1 is slightly greater than φ_2. But if we neglect I_0 as compared to I_2' as in Figure (15-c), then $\varphi_1 = \varphi_2$. Moreover, under this assumption

$N_1I_2' = N_2I_2 = N_1I_1$

$$\therefore \quad \frac{I_2'}{I_2} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = K$$

It shows that under full-load conditions, the ratio of primary and secondary currents is constant.

Example 9: A single-phase transformer has 1000 turns on the primary and 200 turns on the secondary. The no load current is 3 amp. at a p.f. of 0.2 lagging. Calculate the primary current and power-factor when the secondary current is 280 amp at a p.f. of 0.80 lagging.

Solution:

V_2 is taken as reference. $\cos^{-1} 0.80 = 36.87°$

$I_2 = 280 \angle -36.87°$ amp

$I'_2 = (280/5) \angle -36.87°$ amp

$\varphi = \cos^{-1} 0.20 = 78.5°$, $\sin \varphi = 0.98$

$I_1 = I_0 + I'_2$

$= 3(0.20 - j\,0.98) + 56\,(0.80 - j\,0.60)$

$= 0.6 - j\,2.94 + 44.8 - j\,33.6$

$= 45.4 - j\,2.94 + 44.8 - j\,33.6$

$= 45.4 - j\,36.54 = 58.3 \angle 38.86°$

Thus I lags behind the supply voltage by an angle of 38.86°.

Example 10: A single-phase transformer with a ratio of 440/110-V takes a no-load current of 5A at 0.2 power factor lagging. If the secondary supplies a current of 120 A at a p.f. of 0.8 lagging, estimate the current taken by the primary.

Solution:

$\cos \varphi_2 = 0.8$, $\varphi_2 = \cos^{-1}(0.8) = 36°54'$

$\cos \varphi_0 = 0.2$ $\therefore\ \varphi_0 = \cos^{-1}(0.2) = 78°\ 30'$

Now $K = V_2/V_1 = 110/440 = 1/4$

$\therefore\ I_2' = KI_2 = 120 \times 1/4 = 30$ A

$I_0 = 5$ A.

Angle between I_0 and $I_2' = 78°\ 30' - 36°\ 54' = 41°\ 36'$

Using parallelogram law of vectors (Figure (16)) we get,

$I_1 = \sqrt{(5^2 + 30^2 + 2 \times 5 \times 30 \times \cos 41°\ 36)} =$ **34.45 A**

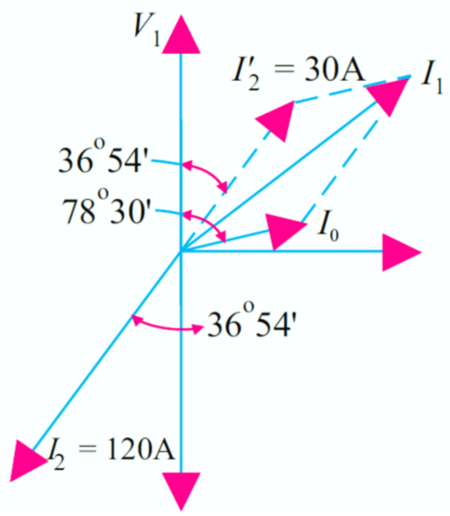


Figure (16)

The resultant current could also have been found by resolving I_2' and I_0 into their X and Y -components.

Note: vectors can be solved byany one of these methods,
1) Parallelogram method (as in example10).
2) Vector components analysis.
3) Complex number methods.

Example 11: A transformer has a primary winding of 800 turns and a secondary winding of 200 turns. When the load current on the secondary is 80 A at 0.8 power factor lagging, the primary current is 25 A at 0.707 power factor lagging. Determine graphically or otherwise the no-load current of the transformer and its phase with respect to the voltage.

Solution:

Here $K = 200/800 = 1/4$; $I_2' = 80 \times (1/4) = 20$ A

$\varphi_2 = \cos^{-1}(0.8) = 36.9°$; $\varphi_1 = \cos^{-1}(0.707) = 45°$

As seen from Figure (17), I_1 is the vector sum of I_0 and I_2'. Let I_0 lag behind V_1 by an angle φ_0.

$I_0 \cos\varphi_0 + 20\cos 36.9° = 25\cos 45°$

$\therefore\ I_0 \cos\varphi_2 = 25 \times 0.707 - 20 \times 0.8 = 1.675$ A

$I_0 \sin\varphi_0 + 20\sin 36.9° = 25\sin 45°$

$\therefore\ I_0 \sin\varphi_0 = 25 \times 0.707 - 20 \times 0.6 = 5.675$ A

$\therefore\ \tan\varphi_0 = 5.675/1.675 = 3.388$

$\therefore\ \varphi_0 = \mathbf{73.3°}$

Now, $I_0 \sin\varphi_0 = 5.675$

$\therefore\ I_0 = 5.675/\sin 73.3° = \mathbf{5.93\ A}$

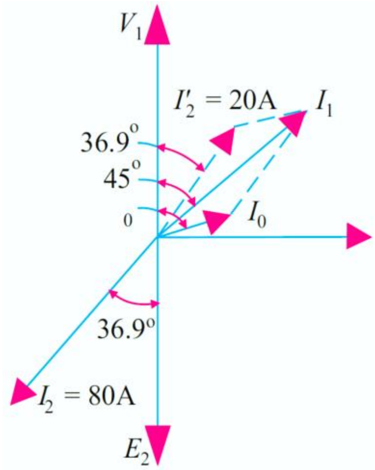


Figure (17)

Example 12: A single phase transformer takes 10 A on no load at p.f. of 0.2 lagging. The turns ratio is 4 : 1 (step down). If the load on the secondary is 200 A at a p.f. of 0.85 lag. Find the primary current and power factor. Neglect the voltage-drop in the winding.

Solution:

Secondary load of 200 A, 0.85 lag is reflected as 50 A, 0.85 lag in terms of the primary equivalent current.

$I_0 = 10\angle{-\varphi_0}$, where $\varphi_0 = \cos^{-1} 0.20 = 78.5°$ lagging

$= 2 - j\,9.8$ amp

$I'_2 = 50\angle{-\varphi_L}$, where $\varphi_L = \cos^{-1} 0.85 = 31.8°$, lagging

$I'_2 = 42.5 - j\,26.35$

Hence primary current,

$I_1 = I_0 + I_2'$

$= 2 - j\,9.8 + 42.5 - j\,26.35$

$= 44.5 - j\,36.15$

$|I_1| = 57.333$ amp, $\varphi = 0.776$ Lag.

$\varphi = \cos^{-1}(44.5 / 57.333) = 39.10°$ lagging

The phasor diagram is shown in Figure (18).

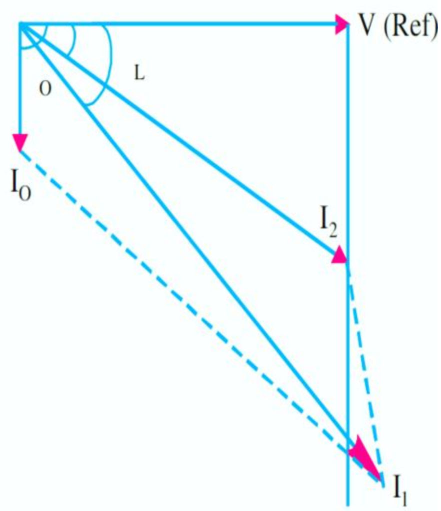


Figure (18)

Tutorial Problems (2)

[1] The primary of a certain transformer takes 1 A at a power factor of 0.4 when it is connected across a 200-V, 50-Hz supply and the secondary is on open circuit. The number of turns on the primary is twice that on the secondary. A load taking 50 A at a lagging power factor of 0.8 is now connected across the secondary. What is now the value of primary current ? **[25.9 A]**

[2] The number of turns on the primary and secondary windings of a single-phase transformer are 350 and 38 respectively. If the primary winding is connected to a 2.2 kV, 50-Hz supply, determine
(a) the secondary voltage on no-load,
(b) the primary current when the secondary current is 200 A at 0.8 p.f. lagging, if the no-load current is 5 A at 0.2 p.f. lagging,
(c) the power factor of the primary current. **[239 V; 25.65 A; 0.715 lag]**

[3] A 400/200-V, 1-phase transformer is supplying a load of 25 A at a p.f. of 0.866 lagging. On no-load the current and power factor are 2 A and 0.208 respectively. Calculate the current taken from the supply. **[13.9 A lagging V1 by 36.1°]**

[4] A transformer takes 10 A on no-load at a power factor of 0.1. The turn ratio is 4 : 1 (step down). If a load is supplied by the secondary at 200 A and p.f. of 0.8, find the primary current and power factor (internal voltage drops in transformer are to be ignored). **[57.2 A; 0.717 lagging]**

[5] A 1-phase transformer is supplied at 1,600 V on the h.v. side and has a turn ratio of 8 : 1. The transformer supplies a load of 20 kW at a power factor of 0.8 lag and takes a magnetising current of 2.0 A at a power factor of 0.2. Calculate the magnitude and phase of the current taken from the h.v. supply. **[17.15 A ; 0.753 lag]**

[6] A 2,200/200-V, transformer takes 1 A at the H.T. side on no-load at a p.f. of 0.385 lagging. Calculate the iron losses. If a load of 50 A at a power of 0.8 lagging is taken from the secondary of the transformer, calculate the actual primary current and its power factor. **[847 W; 5.44 A; 0.74 lag]**

[7] A 400/200-V, I-phase transformer is supplying a load of 50 A at a power factor of 0.866 lagging. The no-load current is 2 A at 0.208 p.f. lagging. Calculate the primary current and primary power factor. **[26.4 A; 0.838 lag]**

10. Transformer with Winding Resistance but No Magnetic Leakage

An ideal transformer was supposed to possess no resistance, but in an actual transformer, there is always present some resistance of the primary and secondary windings. Due to this resistance, there is some voltage drop in the two windings. The result is that:

i) The secondary terminal voltage V_2 is vectorially less than the secondary induced e.m.f. E_2 by an amount I_2 R_2 where R_2 is the resistance of the secondary winding. Hence, V_2 is equal to the vector difference of E_2 and resistive voltage drop I_2 R_2.

$\therefore \overrightarrow{V_2} = \overrightarrow{E_2} - \overrightarrow{I_2 R_2}$ ...vector difference

(ii) Similarly, primary induced e.m.f. E_1 is equal to the vector difference of V_1 and I_1 R_1 where R_1 is the resistance of the primary winding.

$\overrightarrow{E_1} = \overrightarrow{V_1} - \overrightarrow{I_1 R_1}$ ...vector difference

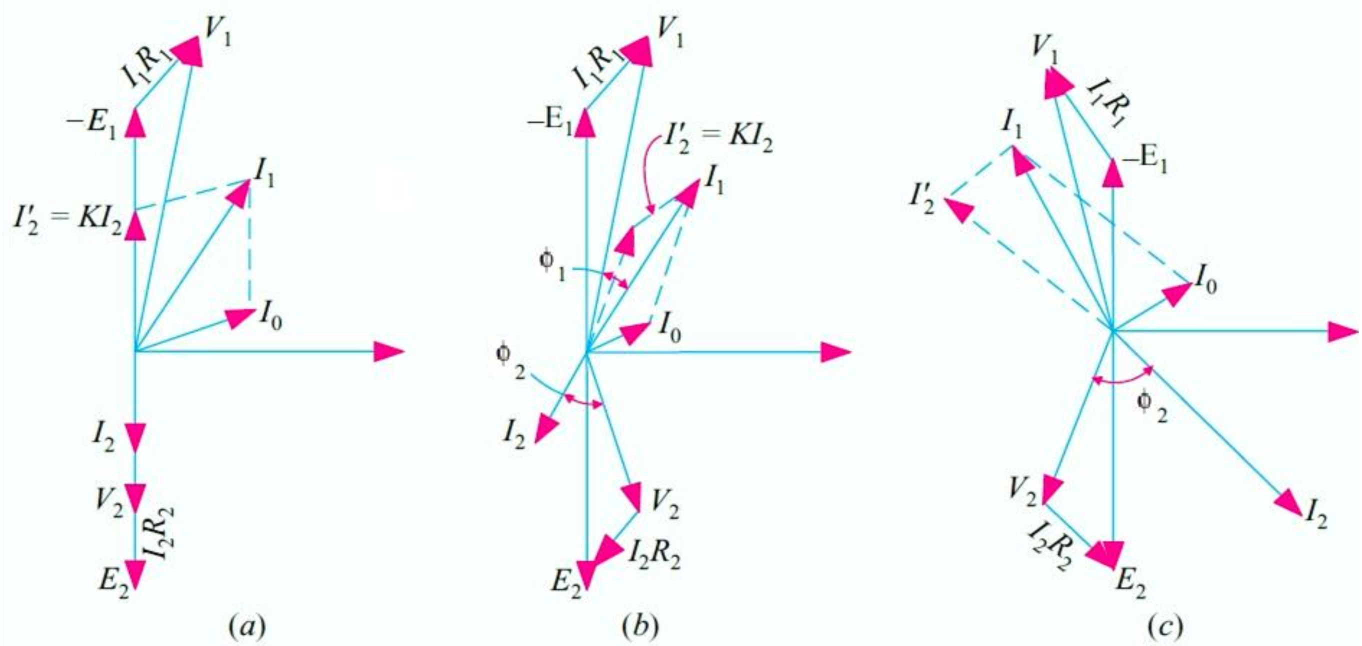


Figure (19)

The vector diagrams for non-inductive, inductive and capacitive loads are shown in Figure (19) (a), (b) and (c) respectively.

11. Equivalent Resistance

In Figure (20) a transformer is shown whose primary and secondary windings have resistances of R_1 and R_2 respectively. It would now be shown that the resistances of the two windings can be transferred to any one of the two windings. The advantage of concentrating both the resistances in one winding is that it makes calculations very simple and easy because one has then to work in one winding only.

It is to be noted that

1) a resistance of R_1 in primary is equivalent to K^2R_1 in secondary. Hence, it is called *equivalent primary resistance as referred to secondary* i.e. R'_1 (as shown in Figure (21)).

2) a resistance of R_2 in secondary is equivalent to R_2/K^2 in primary. Hence, it is called the *equivalent secondary resistance as referred to primary* i.e. R_2' (as shown in Figure (20)).

3) *Total or effective resistance of the transformer as referred to primary* is,

R_{01} = primary resistance + equivalent secondary resistance as referred to primary

$= R_1 + R_2' = R_1 + R_2/K^2$

4) Similarly, total *transformer resistance as referred to secondary* is,

R_{02} = secondary resistance + equivalent primary resistance as referred to secondary

$= R_2 + R_1' = R_2 + K^2R_1$

5) The copper loss in secondary is $(I_2)^2R_2$. This loss is supplied by primary which takes a current of I_1. Hence if R_2' is the *equivalent resistance in primary which would have caused the same loss* as R_2 in secondary, then, $I_1^2R_2' = I_2^2R_2$ or $R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2$

Now, if **we neglect no-load current** I_0, then $I_2/I_1 = 1/K$. Hence, $R_2' = R_2/K_2$. Similarly, equivalent primary resistance as referred to secondary is $R_1' = K_2R_1$

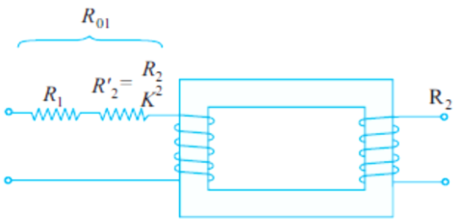


Figure (20)

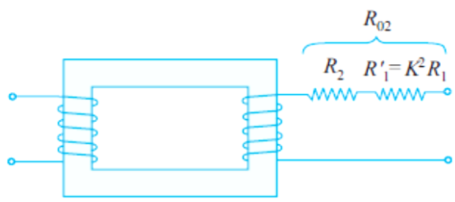


Figure (21)

6) When shifting any **voltage** from one winding to another only **K** is used,

$E'_1 = KE_1$ & $E'_2 = E_2/K$

7) When shifting any **current** from one winding to another only **K** is used but inverse to the transformation of resistances and currents,

$I'_1 = I_1/K$ & $I'_2 = KI_2$

8) It is important to remember that,

When shifting any primary resistance to the secondary, **multiply** it by K^2 i.e. (transformation ratio)^2. When shifting secondary resistance to the primary, **divide** it by K^2.

The result can be cross-checked by another approach. The high voltage winding is always low current winding and hence the resistance of high voltage side is high. The low voltage side is high current side and hence resistance of low voltage side is low. So while transferring resistance from low voltage side to high voltage side, its value must increase while transferring resistance from high voltage side to low voltage side, its value must decrease.

High voltage side →	Low current side →	High resistance side
Low voltage side →	High current side →	Low resistance side

12. Magnetic Leakage

In the preceding discussion, it has been assumed that all the flux linked with primary winding also links the secondary winding. But, in practice, it is impossible to realize this condition. It is found, however, that all the flux linked with primary does not link the secondary but part of it *i.e.* Φ_{L1} completes its magnetic circuit by passing through air rather than around the core, as shown in Figure (22). This leakage flux is produced when the m.m.f. due to primary ampere-turns existing between points *a* and *b*, acts along the leakage paths. Hence, this flux is known as ***primary leakage flux*** and is proportional to the primary ampere-turns alone because the secondary turns do not link the magnetic circuit of Φ_{L1}.The flux Φ_{L1} is in time phase with I_1. It induces an e.m.f. e_{L1} in primary but not in secondary.

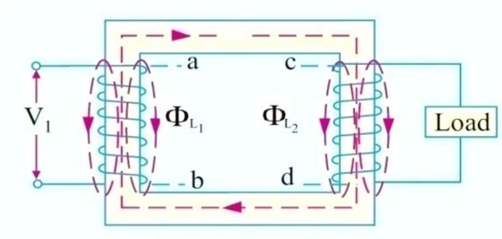


Figure (22)

Similarly, secondary ampere-turns (or m.m.f.) acting across points *c* and *d* set up leakage flux Φ_{L2} which is linked with secondary winding alone (and not with primary turns). This flux Φ_{L2} is in time phase with I_2 and produces a self-induced e.m.f. e_{L2} in secondary (but not in primary).

At no load and light loads, the primary and secondary ampere-turns are small, hence leakage fluxes are negligible. But when load is increased, both primary and secondary windings carry huge currents. Hence, large m.m.f.s are set up which, while acting on leakage paths, increase the leakage flux.

As said earlier, the leakage flux linking with each winding, produces a self-induced e.m.f. in that winding. Hence, in effect, it is equivalent to a small choker or inductive coil in series with each winding such that voltage drop in each series coil is equal to that produced by leakage flux. In other words, ***a transformer with magnetic leakage is equivalent to an ideal transformer with inductive coils connected in both primary and secondary circuits*** as shown in Figure (23) such that the internal e.m.f. in each inductive coil is equal to that due to the corresponding leakage flux in the actual transformer.

$X_1 = e_{L1}/I_1$ and $X_2 = e_{L2}/I_2$

The terms X_1 and X_2 are known as primary and secondary *leakage* reactances respectively.

Following few points should be kept in mind :

1) The leakage flux links one or the other winding but ***not both***, hence it in no way contributes to the transfer of energy from the primary to the secondary winding.

2) The primary voltage V_1 will have to supply reactive drop I_1X_1 in addition to I_1R_1. Similarly E_2 will have to supply $I_2 R_2$ and $I_2 X_2$.

3) In an actual transformer, the primary and secondary windings are not placed on separate legs or limbs as shown in Figure (23) because due to their being widely separated, large primary and secondary leakage fluxes would result. These leakage fluxes are minimised by sectionalizing and interleaving the primary and secondary windings.

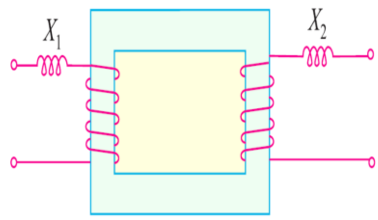


Figure (23)

13. Transformer with Resistance and Leakage Reactance

In Figure (24) the primary and secondary windings of a transformer with reactances taken out of the windings are shown. The primary impedance is given by

$Z_1 = \sqrt{R_1^2 + X_1^2}$

Similarly, secondary impedance is given by

$Z_2 = \sqrt{R_2^2 + X_2^2}$

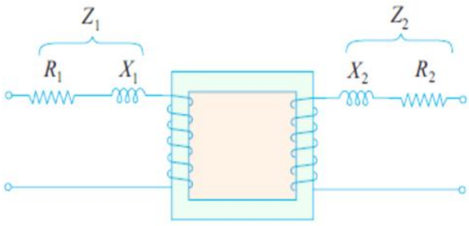


Figure (24)

The resistance and leakage reactance of each winding is responsible for some voltage drop in each winding. In primary, the leakage reactance drop is I_1X_1 (usually 1 or 2% of V_1). Hence

$\mathbf{V_1} = \mathbf{E_1} + \mathbf{I_1}(R_1 + jX_1) = \mathbf{E_1} + \mathbf{I_1}\,\mathbf{Z_1}$

Similarly, there are I_2R_2 and I_2X_2 drops in secondary which combine with V_2 to give E_2.

$\mathbf{E_2} = \mathbf{V_2} + \mathbf{I_2}(R_2 + jX_2) = \mathbf{V_2} + \mathbf{I_2}\,\mathbf{Z_2}$

The vector diagram for such a transformer for different kinds of loads is shown in Figure (25). In these diagrams, vectors for resistive drops are drawn parallel to current vectors whereas reactive drops are perpendicular to the current vectors. The angle φ_1 between V_1 and I_1 gives the power factor angle of the transformer.

It may be noted that leakage reactances can also be transferred from one winding to the other in the same way as resistance.

∴ $X_2' = X_2/K^2$ and $X_1' = K^2X_1$

and $X_{01} = X_1 + X_2' = X_1 + X_2/K^2$ and $X_{02} = X_2 + X_1' = X_2 + K^2X_1$

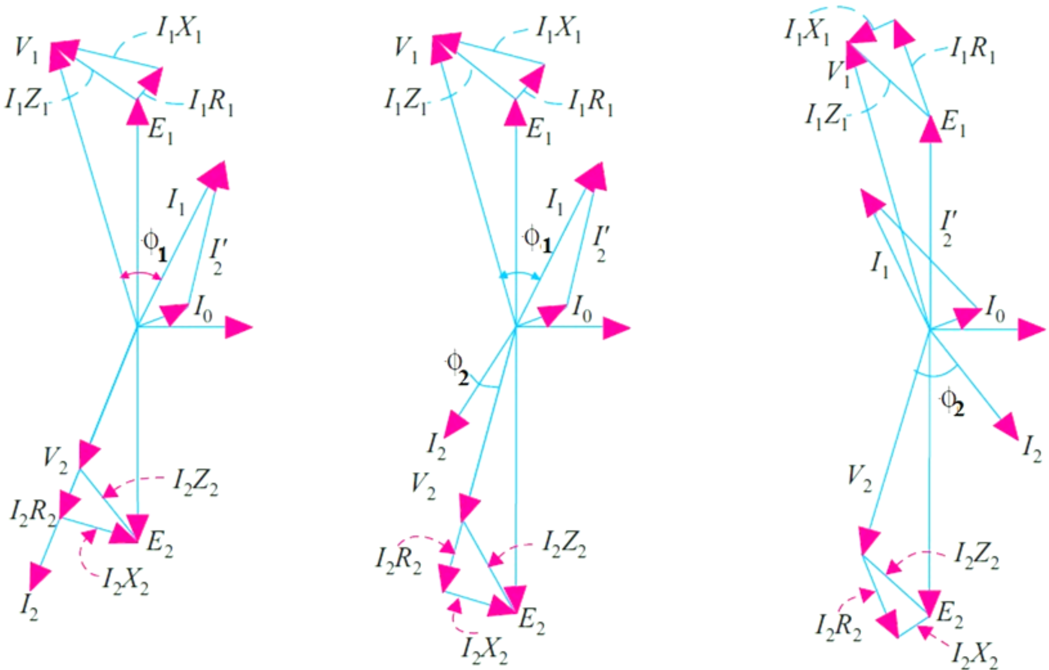
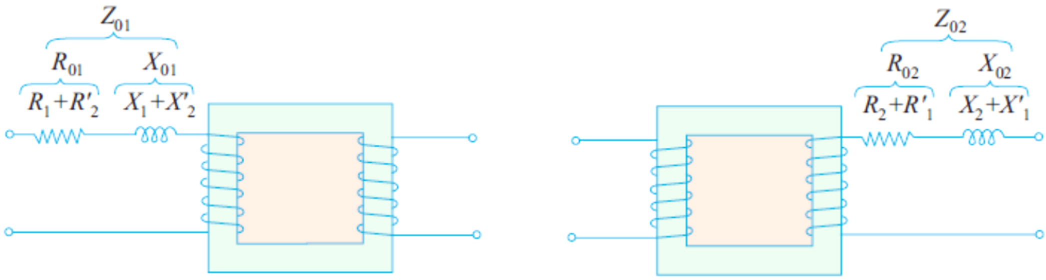


Figure (25)



(a) (b)

Figure (26)

It is obvious that total impedance of the transformer as referred to primary is given by

$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$ …Figure (26-a)

$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$ …Figure (26-b)

Example 13: A 30 kVA, 2400/120-V, 50-Hz transformer has a high voltage winding resistance of 0.1 Ω and a leakage reactance of 0.22Ω. The low voltage winding resistance is 0.035 Ω and the leakage reactance is 0.012 Ω. Find the equivalent winding resistance, reactance and impedance referred to the **(i)** high voltage side and **(ii)** the low-voltage side.

Solution:

K = 120/2400 = 1/20; $R_1 = 0.1\ \Omega$, $X_1 = 0.22\ \Omega$, $R_2 = 0.035\ \Omega$ and $X_2 = 0.012\ \Omega$

(i) Here, high-voltage side is, obviously, the primary side. Hence, values as referred to primary sideare;

$R_{01} = R_1 + R_2' = R_1 + R_2/K^2 = 0.1 + 0.035/(1/20)^2 =$ **14.1 Ω**

$X_{01} = X_1 + X_2' = X_1 + X_2/K^2 = 0.22 + 0.12/(1/20)^2 =$ **5.02 Ω**

$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{14.1^2 + 5.02^2} =$ **15 Ω**

(ii) $R_{02} = R_2 + R_1' = R_2 + K^2R_1 = 0.035 + (1/20)^2 \times 0.1 =$ **0.03525 Ω**

$X_{02} = X_2 + X_1' = X_2 + K^2X_1 = 0.012 + (1/20)^2 \times 0.22 =$ **0.01255 Ω**

$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{0.0325^2 + 0.01255^2} =$ **0.0374 Ω**

(or $Z_{02} = K2Z_{01} = (1/20)^2 \times 15 = 0.0375\ \Omega$)

Example 14: A 50-kVA, 4,400/220-V transformer has R_1 = 3.45 Ω, R_2 = 0.009 Ω. The values of reactances are X_1 = 5.2 Ω and X_2 = 0.015 Ω. Calculate for the transformer **(i)** equivalent resistance as referred to primary **(ii)** equivalent resistance as referred to secondary **(iii)** equivalent reactance as referred to both primary and secondary **(iv)** equivalent impedance as referred to both primary and secondary **(v)** total Cu loss, first using individual resistances of the two windings and secondly, using equivalent resistances as referred to each side.

Solution:

Full-load I_1 = 50,000/4,400 = 11.36 A (assuming 100% efficiency)
Full-load I_2 = 50,000/220 = 227 A; K = 220/4,400 = 1/20

(i) $R_{01} = R_1 + \frac{R_2}{K^2} = 3.45 + \frac{0.009}{(1/20)^2} = 3.45 + 3.6 = \mathbf{7.05\ \Omega}$

(ii) $R_{02} = R_2 + K^2R_1 = 0.009 + (1/20)^2 \times 3.45 = 0.009 + 0.0086 = \mathbf{0.0176\ \Omega}$
Also, $R_{02} = K^2 R_{01} = (1/20)^2 \times 7.05 = 0.0176\ \Omega$ (check)

(iii) $X_{01} = X_1 + X_2' = X_1 + X_2/K^2 = 5.2 + 0.015/(1/20)^2 = \mathbf{11.2\ \Omega}$
$X_{02} = X_2 + X_1' = X_2 + K^2 X_1 = 0.015 + 5.2/20^2 = \mathbf{0.028\ \Omega}$
Also $X_{02} = K^2 X_{01} = 11.2/400 = 0.028\ \Omega$ (check)

(iv) $Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{7.05^2 + 11.2^2} = \mathbf{13.23\ \Omega}$
$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{0.0176^2 + 0.028^2} = \mathbf{0.03311\ \Omega}$
Also $Z_{02} = K^2Z_{01} = 13.23/400 = 0.0331\ \Omega$ (check)

(v) Cu loss $= I_1^2 R_1 + I_2^2 R_2 = 11.36^2 \times 3.45 + 227^2 \times 0.009 = \mathbf{910\ W}$
Also Cu loss $= I_1^2R_{01} = 11.36^2 \times 7.05 = \mathbf{910\ W}$
$= I_2^2R_{02} = 227^2 \times 0.0176 = \mathbf{910\ W}$

Example 15: A transformer with a 10 : 1 ratio and rated at 50-kVA, 2400/240-V, 50-Hz is used to step down the voltage of a distribution system. The low tension voltage is to be kept constant at 240 V. **(a)** What load impedance connected to low-tension size will be loading the transformer fully at 0.8 power factor (lag) ? **(b)** What is the value of this impedance referred to high tension side ? **(c)** What is the value of the current referred to the high tension side ?

Solution:

(a) F. L. I_2 = 50,000/240 = 625/3 A; $Z_2 = \frac{240}{(625/3)} = \mathbf{1.152\ \Omega}$

(b) K = 240/2400 = 1/10
The secondary impedance referred to primary side is,
$Z_2' = Z_2/K^2 = 1.142/(1/10)^2 = \mathbf{114.2\ \Omega}$

(c) Secondary current referred to primary side is $I_2' = KI_2 = (1/10) \times (625/3) = \mathbf{20.83\ A}$

Example 16: The following data refer to a I-phase transformer :
Turn ratio 19.5 : 1 ; $R_1 = 25\ \Omega$; $X_1 = 100\ \Omega$; $R_2 = 0.06\ \Omega$; $X_2 = 0.25\ \Omega$. No-load current = 1.25 A leading the flux by 30°.
The secondary delivers 200 A at a terminal voltage of 500 V and p.f. of 0.8 lagging. Determine by the aid of a vector diagram, the primary applied voltage, the primary p.f. and the efficiency.

Solution:

The vector diagram is similar to Figure (25) which has been redrawn as Figure (27). Let us take V_2 as the reference vector.

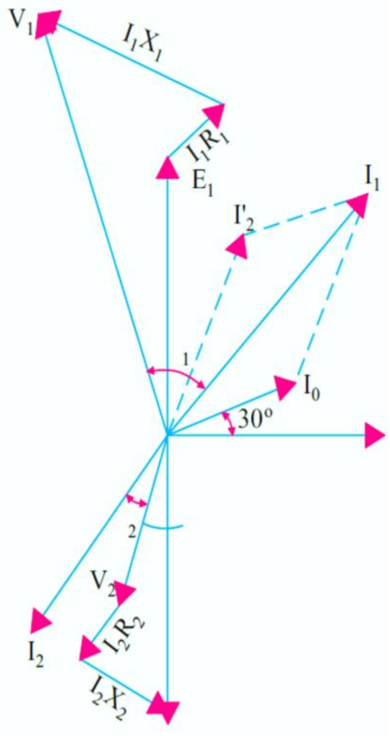


Figure (27)

$\therefore\ \mathbf{V_2} = 500 \angle 0° = 500 + j0$

$\mathbf{I_2} = 200\,(0.8 - j\,0.6) = 160 - j\,120$

$\mathbf{Z_2} = (0.06 + j\,0.25)$

$\mathbf{E_2} = V_2 + \mathbf{I_2}\,\mathbf{Z_2}$

$= (500 + j\,0) + (160 - j\,120)\,(0.06 + j\,0.25)$

$= 500 + (39.6 + j\,32.8) = 539.6 + j\,32.8 = 541 \angle 3.5°$

Obviously, $\beta = 3.5°$

$\mathbf{E_1} = \mathbf{E_2}/K = 19.5\,\mathbf{E_2} = 19.5\,(539.6 + j\,32.8) = 10{,}520 + j\,640$

$\therefore\ -\mathbf{E_1} = -10{,}520 - j\,640 = 10{,}540 \angle 183.5°$

$\mathbf{I_2'} = -\mathbf{I_2}K = (-160 + j\,120)/19.5 = -8.21 + j\,6.16$

As seen from Figure (27), $\mathbf{I_0}$ leads $\mathbf{V_2}$ by an angle,

$= 3.5° + 90° + 30° = 123.5°$

$\therefore\ \mathbf{I_0} = 1.25 \angle 123.5°$

$= 1.25\,(\cos 123.5° + j \sin 123.5°)$

$= 1.25\,(-\cos 56.5° + j \sin 56.5°)$

$= -0.69 + j\,1.04$

$\mathbf{I_1} = \mathbf{I_2'} + \mathbf{I_0} = (-8.21 + j\,6.16) + (-0.69 + j\,1.04) = -8.9 + j\,7.2 = 11.45 \angle 141°$

$\mathbf{V_1} = -\mathbf{E_1} + \mathbf{I_1}\mathbf{Z_1}$

$= -10{,}520 - j\,640 + (-8.9 + j\,7.5)\,(25 + j\,100) = -10{,}520 - j\,640 - 942 - j\,710$

$= -11{,}462 - j\,1350 = 11{,}540 \angle 186.7°$

Phase angle between $\mathbf{V_1}$ and $\mathbf{I_1}$ is $= 186.7° - 141° = 45.7°$

$\therefore$ primary p.f. = cos 45.7° = **0.698 (lag)**

No-load primary input power = $V_1 I_0 \cos \varphi_0 = 11{,}540 \times 1.25 \times \cos 60° = 7{,}210$ **W**

$R_{02} = R_2 + K^2 R_1 = 0.06 + 25/19.5^2 = 0.1257\ \Omega$

Total Cu loss as referred to secondary = $I_2^2 R_{02} = 200^2 \times 0.1257 = 5{,}030$ W

Output = $V_2 I_2 \cos \varphi_2 = 500 \times 200 \times 0.8 = 80{,}000$ W

Total losses = 5030 + 7210 = 12,240 W,

Input = 80,000 + 12,240 = 92,240 W

Or Input = $V_1 I_1 \cos\varphi_1 = 11540 \times 11.45 \times \cos(186.7 - 141) \cong 92240$ W

$\eta = 80{,}000/92{,}240 = 0.8674$ or **86.74%**

Example 17: A 100 kVA, 1100/220 V, 50 Hz, single-phase transformer has a leakage impedance of (0.1 + 0/40) ohm for the H.V. winding and (0.006 + 0.015) ohm for the L.V. winding. Find the equivalent winding resistance, reactance and impedance referred to the H.V. and L.V. sides.

Solution:

Turns ratio = $(N_1/N_2) = (V_1/ V_2) = 1100/220 = 5$

(i) Referred to H.V. side :

Resistance = $r_1 + r_2 = 0.1 + (25 \times 0.006) = 0.25$ ohm

Reactance = $x_1 + x_2' = 0.4 + (25 \times 0.015) = 0.775$ ohm

Impedance = $(0.25^2 + 0.775^2)^{0.5} = 0.8143$ ohm

(ii) Referred to L.V. side :

Resistance = 0.25/25 = 0.01

(or resistance = 0.006 + (0.1/25) = 0.01 ohm)

Reactance = 0.775/25 = 0.031 ohm

Impedance = 0.8143/25 = 0.0326 ohm

14. Total Approximate Voltage Drop in a Transformer

When the transformer is on no-load, then V_1 is approximately equal to E_1. Hence $E_2 = KE_1 = KV_1$. Also, $E_2 = {}_0V_2$ where ${}_0V_2$ is secondary terminal voltage on **no load**, hence no-load secondary terminal voltage is KV_1. The secondary voltage on load is V_2. The difference between the two is $I_2 Z_{02}$ as shown in Figure (28). The approximate voltage drop of the transformer **as referred to secondary** is found thus:

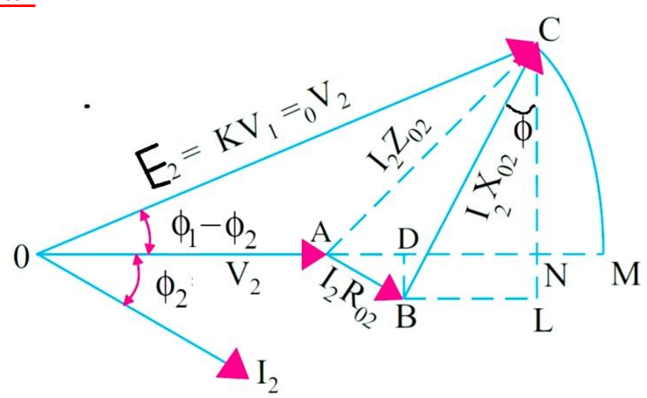


Figure (28)

With O as the centre and radius OC draw an arc cutting OA produced at M. The total voltage drop I_2Z_{02} = AC = AM which is approximately equal to AN. From B draw BD perpendicular on OA produced. Draw CN perpendicular to OM and draw BL parallel to OM.

Approximate voltage drop

$$= AN = AD + DN = \boxed{I_2R_{02} \cos \varphi + I_2X_{02} \sin \varphi}$$

Where $\varphi_1 = \varphi_2 = \varphi$ (approx).

This is the value of approximate voltage drop for a **lagging** power factor.

The different figures for unity and leading power factors are shown in Figure (29) (a) and (b) respectively.

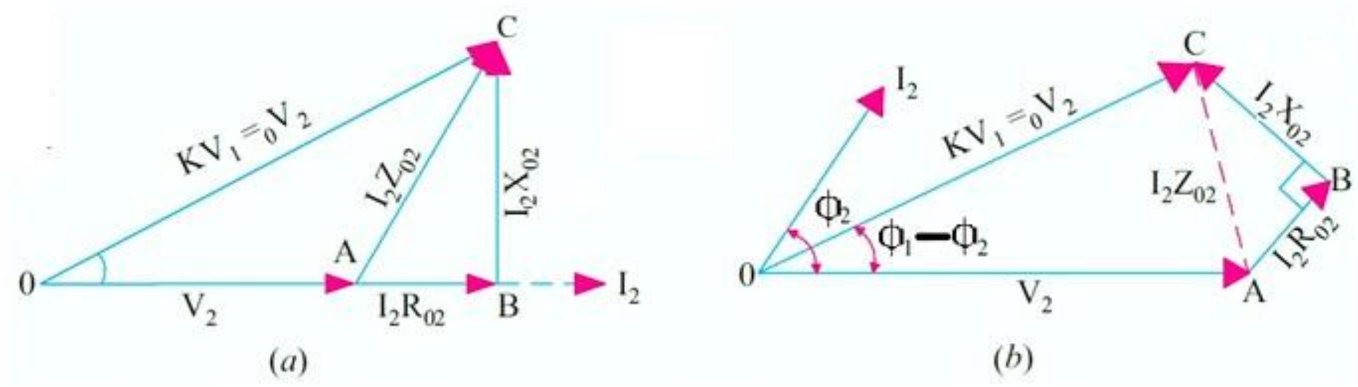


Figure (29)

The approximate voltage drop for **leading** power factor becomes,

$$= \boxed{(I_2R_{02} \cos \varphi - I_2 X_{02} \sin \varphi)}$$

In general, approximate voltage drop is,

$$= \boxed{(I_2 R_{02} \cos \varphi \pm I_2 X_{02} \sin \varphi)}$$

It may be noted that approximate voltage drop as referred to primary is,

$$= \boxed{(I_1R_{01} \cos \varphi \pm I_1 X_{01} \sin \varphi)}$$

% voltage drop in secondary is,

$$= \frac{I_2R_{02}\cos\emptyset \pm I_2X_{02}\sin\emptyset}{{}_0V_2} \times 100$$

$$= \frac{100 \times I_2R_{02}}{{}_0V_2}\cos\emptyset \pm \frac{100 \times I_2X_{02}}{{}_0V_2}\sin\emptyset$$

$$= v_r\cos\varphi \pm v_x\sin\varphi$$

Where $v_r = \frac{100 \times I_2R_{02}}{{}_0V_2}$ = percentage resistive drop $= \frac{100 \times I_1R_{01}}{V_1}$

$v_x = \frac{100 \times I_2X_{02}}{{}_0V_2}$ = percentage reactive drop $= \frac{100 \times I_1X_{01}}{V_1}$

15. Exact Voltage Drop

With reference to Figure (28), it is to be noted that exact voltage drop is *AM* and not *AN*. If we add the quantity *NM* to *AN*, we will get the exact value of the voltage drop. Considering the right-angled triangle *OCN*, we get,

$$NC^2 = OC^2 - ON^2 = (OC + ON)(OC - ON) = (OC + ON)(OM - ON) = 2\,OC \times NM$$

$\therefore\ NM = NC^2/2OC$ Now, $NC = LC - LN = LC - BD$

$\therefore\ NC = I_2 X_{02}\cos\varphi - I_2 R_{02}\sin\varphi$

$$\therefore NM = \frac{(I_2X_{02}\cos\varphi - I_2R_{02}\sin\varphi)^2}{2_0V_2}$$

$\therefore$ For a ***lagging*** power factor, exact voltage drop is

$$= AN + NM = \boxed{(I_2R_{02}\cos\varphi + I_2X_{02}\sin\varphi) + \frac{(I_2X_{02}\cos\varphi - I_2R_{02}\sin\varphi)^2}{2_0V_2}}$$

$\therefore$ For a ***leading*** power factor, the expression becomes

$$= \boxed{(I_2R_{02}\cos\varphi - I_2X_{02}\sin\varphi) + \frac{(I_2X_{02}\cos\varphi + I_2R_{02}\sin\varphi)^2}{2_0V_2}}$$

In general, the voltage drop is,

$$= \boxed{(I_2R_{02}\cos\varphi \pm I_2X_{02}\sin\varphi) + \frac{(I_2X_{02}\cos\varphi \mp I_2R_{02}\sin\varphi)^2}{2_0V_2}}$$

Percentage drop is,

$$= \frac{(I_2R_{02}\cos\varphi \pm I_2X_{02}\sin\varphi)\times 100}{0V_2} + \frac{(I_2X_{02}\cos\varphi \mp I_2R_{02}\sin\varphi)^2\times 100}{2_0V_2^2}$$

$$= (v_r\cos\varphi \pm v_x\sin\varphi) + \left(\frac{1}{200}\right)(v_x\cos\varphi \mp v_r\sin\varphi)^2$$

The upper signs are to be used for a ***lagging*** power factor and the lower ones for a ***leading*** power factor.

Example 18: A 230/460-V transformer has a primary resistance of 0.2 Ω and reactance of 0.5 Ω and the corresponding values for the secondary are 0.75Ω and 1.8Ω respectively. Find the secondary terminal voltage when supplying 10 A at 0.8 p.f. lagging.

Solution:

$K = 460/230 = 2$

$R_{02} = R_2 + K^2 R_1$

$= 0.75 + 2^2 \times 0.2 = 1.55\ \Omega$

$X_{02} = X_2 + K^2 X_1$

$= 1.8 + 2^2 \times 0.5 = 3.8\ \Omega$

Voltage drop $= I_2 (R_{02} \cos\varphi + X_{02} \sin\varphi)$

$= 10 (1.55 \times 0.8 + 3.8 \times 0.6) = 35.2\text{V}$

∴ Secondary terminal voltage = 460 − 35.2 = **424.8 V**

Example 19: Calculate the regulation of a transformer in which the percentage resistance drops is 1.0% and percentage reactance drop is 5.0% when the power factor is **(a)** 0.8 lagging **(b)** unity and **(c)** 0.8 leading.

Solution:

We will use the approximate expression .

(a) p.f. = cos φ = 0.8 lag

$\mu = v_r \cos\varphi + v_x \sin\varphi$

$= 1 \times 0.8 + 5 \times 0.6 =$ **3.8%**

(b) p.f. = cos φ = 1

$\mu = 1 \times 1 + 5 \times 0 =$ **1%**

(c) p.f. = cos φ = 0.8 lead

$\mu = 1 \times 0.8 - 5 \times 0.6 =$ **− 2.2%**

16. Equivalent Circuit

The transformer shown diagrammatically in Figure (30-a) can be resolved into an equivalent circuit in which the resistance and leakage reactance of the transformer are imagined to be external to the winding whose only function then is to transform the voltage [Figure (30-b)].

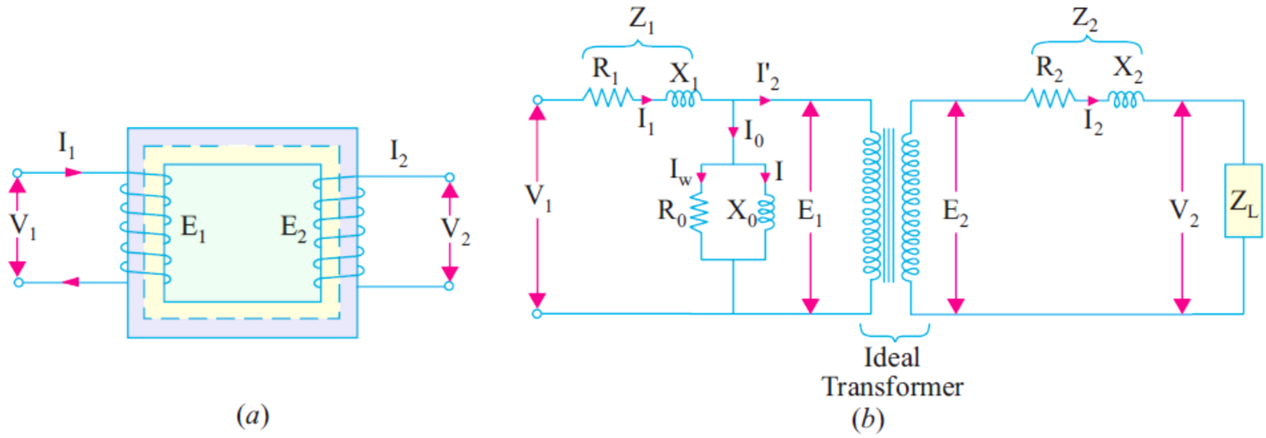


Figure (30)

The no-load current I_0 is simulated by pure inductance X_0 taking the magnetising component I_μ and a non-inductive resistance R_0 taking the working component I_w connected in parallel across the primary circuit. The value of E_1 is obtained by subtracting vectorially $I_1 Z_1$ fromV_1. The value of $X_0 = E_1/I_\mu$ and of $R_0 = E_1/I_w$. It is clear that E_1 and E_2 are related to each other by expression

$E_2/E_1 = N_2/N_1 = K$.

To make transformer calculations simpler, it is preferable to transfer voltage, current and impedance either to the primary or to the secondary. In that case, we would have to work in one winding only which is more convenient.

The primary equivalent of the secondary induced voltage is $E_2' = E_2/K = E_1$.

Similarly, primary equivalent of secondary terminal or output voltage is $V_2' = V_2/K$.

Primary equivalent of the secondary current is $I_2' = KI_2$.

For transferring secondary impedance to primary K^2 is used.

$$R_2' = R_2/K^2,\ X_2' = X_2/K^2,\ Z_2' = Z_2/K^2$$

The total equivalent circuit of the transformer is obtained by adding in the primary impedance as shown in Figure (31). This is known as the exact equivalent circuit but it presents a somewhat harder circuit problem to solve. A simplification can be made by transferring the exciting circuit across the terminals as in Figure (32) or in Figure (33-a). It should be noted that in this case $X_0 = V_1/I_\mu$.

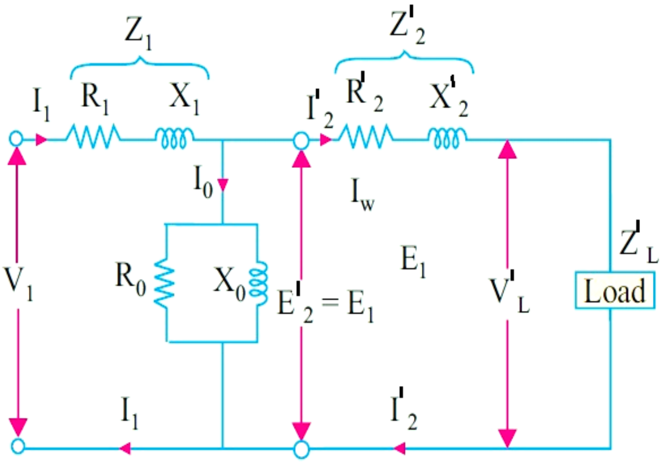


Figure (31): Exact equivalent circuit

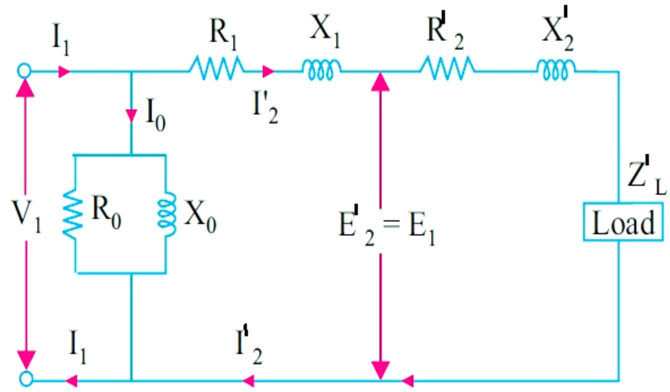


Figure (32): Approximate equivalent circuit

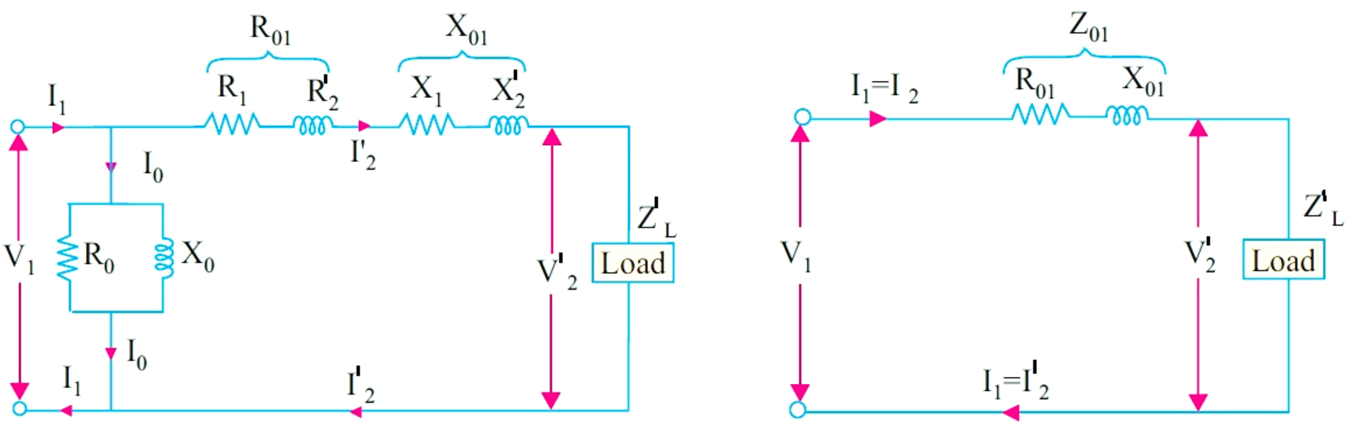
Further simplification may be achieved by omitting I_0 altogether as shown in Figure (35-b). From Figure (31) it is found that total impedance between the input terminal is

$$\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_m \,||\, (\mathbf{Z}_2' + \mathbf{Z}_L') = \left(\mathbf{Z}_1 + \frac{\mathbf{Z}_m\,(\mathbf{Z}_2' + \mathbf{Z}_L')}{\mathbf{Z}_m + (\mathbf{Z}_2' + \mathbf{Z}_L')} \right)$$

where $\mathbf{Z}_2' = R_2' + jX_2'$ and $\mathbf{Z}_m$ = impedance of the exciting circuit.

This is so because there are two parallel circuits, one having an impedance of $\mathbf{Z}_m$ and the other having $\mathbf{Z}_2'$ and $\mathbf{Z}_L'$ in series with each other.

$$\therefore \qquad \mathbf{V}_1 = \mathbf{I}_1 \left[\mathbf{Z}_1 + \frac{\mathbf{Z}_m\,(\mathbf{Z}_2' + \mathbf{Z}_L')}{\mathbf{Z}_m + (\mathbf{Z}_2' + \mathbf{Z}_L')} \right]$$



(a) **(b)**

Figure (33)

Example 20: The parameters of a 2300/230 V, 50-Hz transformer are given below :

$R_1 = 0.286\ \Omega$ $R_2' = 0.319\ \Omega$ $R_0 = 250\ \Omega$

$X_1 = 0.73\ \Omega$ $X_2' = 0.73\ \Omega$ $X_0 = 1250\ \Omega$

The secondary load impedance $Z_L = 0.387 + j\,0.29$. Solve the exact equivalent circuit with normal voltage across the primary.

Solution:

$K = 230/2300 = 1/10;$ $Z_L = 0.387 + j\,0.29$

$\mathbf{Z}_L' = \mathbf{Z}_L/K^2 = 100\,(0.387 + j\,0.29) = 38.7 + j\,29 = 48.4\angle 36.8°$

$\therefore\ \mathbf{Z}_2' + \mathbf{Z}_L' = (38.7 + 0.319) + j(29 + 0.73) = 39.02 + j29.73 = 49.0\angle 37.3°$

$Y_m = (0.004 - j\,0.0008)$

$\mathbf{Z}_m = 1/\mathbf{Y}_m = 240 + j48 = 245\angle 11.3°$

$\mathbf{Z}_m + (\mathbf{Z}_2' + \mathbf{Z}_L') = (240 + j48) + (39 + j29.7) = 290\angle 15.6°$

$$\therefore\ \mathbf{I}_1 = \frac{V_1}{Z_1 + \dfrac{Z_m(Z_2' + Z_L')}{Z_m + (Z_2' + Z_L')}} = \left[\frac{2300\angle 0}{0.286 + j0.73 + 41.4\angle 33°}\right] = \left[\frac{2300\angle 0}{42\angle 33.7°}\right] = 54.8\angle -33.7°$$

Now

$$\mathbf{I}_2' = I_1 \times \frac{Z_m}{Z_m + (Z_2' + Z_L')} = 54.8\angle -33.7° \times \frac{245\angle 11.3°}{290\angle 15.6°} = 54.8\angle -33.7° \times 0.845\angle -4.3°$$

$$= 46.2\angle -38°$$

$$\mathbf{I}_0 = I_1 \times \frac{(Z_2' + Z_L')}{Z_m + (Z_2' + Z_L')} = 54.8\angle -33.7° \times 0.169\angle 21.7° = 9.26\angle -12°$$

Input power factor = cos 33.7° = 0.832 lagging

Power input = $V_1 I_1 \cos\varphi_1 = 2300 \times 54.8 \times 0.832 = 105$ Kw

Power output (P_o)= $I_2'^2 R_L' = 46.2^2 \times 38.7 = 82.7$ Kw

Or $P_o = I_2^2(R_L) = (462)^2 \times 0.387 = 82.6$ kW

Or $P_o = V_2' I_2' \cos\varphi_2'$, where $V_2' = V_2 / K = 230 / (230/2300) = 2300\ V = V_1$

$= 2300 \times 46.2 \times \cos(-38) = 83.73$ Kw

Or $P_o = V_2 I_2 \cos\varphi_2$, where $I_2 = I_2' / K = (46.2\angle -38°) / (1/10) = 462\angle -38°$ A

$= 230 \times 462 \times \cos(-38) = 83.73$ Kw

Note: $K = I_2'/I_2 = (I_1 + I_0)/I_2 = I_1/I_2$, because I_0 is negligible.

Primary Cu loss = $I_1^2 R_1 = 54.82 \times 0.286 = 860$ W

Secondary Cu loss = $I_2'^2 R_2' = 46.22 \times 0.319 = 680$ W

Core loss = $9.26^2 \times 240 = 20.6$ kW

$\eta = (82.7/105) \times 100 = 78.8\%$

$V_2' = I_2' Z_L' = 46.2 \times 48.4 = 2{,}240$V

$$\therefore\ \text{Regulation} = \frac{2300 - 2240}{2240} \times 100 = \mathbf{2.7\%}$$

17. Transformer Tests

The performance of a transformer can be calculated on the basis of its equivalent circuit which contains [Figure (33-a)] four main parameters, the equivalent resistance R_{01} as referred to primary (or secondary R_{02}), the equivalent leakage reactance X_{01} as referred to primary (or secondary X_{02}), the core-loss conductance G_0 (or resistance R_0) and the magnetising susceptance B_0 (or reactance X_0). These constants or parameters can be easily determined by two tests:

(i) **open-circuit test** and
(ii) **short circuit test.**

These tests are very economical and convenient, because they furnish the required information without actually loading the transformer. In fact, the testing of very large a.c. machinery consists of running two tests similar to the open and short-circuit tests of a transformer.

(i) Open-circuit or No-load Test

The purpose of this test is to determine no-load loss or core loss and no-load I_0 which is helpful in finding X_0 and R_0.

One winding of the transformer – whichever is convenient but usually high voltage winding – is left open and the other is connected to its supply of normal voltage and frequency. A wattmeter W, voltmeter V and an ammeter A are connected in the low voltage winding i.e. primary winding in the present case.

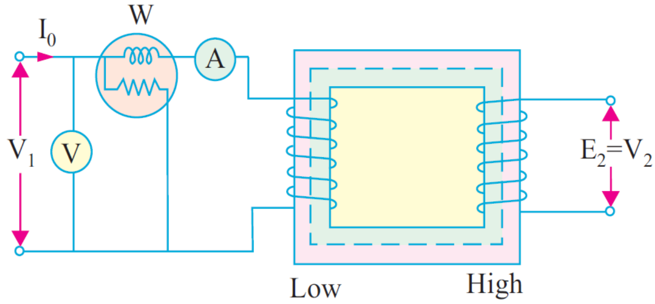


Figure (34)

With normal voltage applied to the primary, normal flux will be set up in the core, hence normal iron losses will occur which is recorded by the wattmeter. As the primary no-load current I_0 (as measured by ammeter) is small (usually 2 to 10% of rated load current), Cu loss is negligibly small in primary and nil in secondary (it being open). Hence, the wattmeter reading represents practically the core loss under no-load condition (and which is the same for all loads).

It should be noted that since I_0 is itself very small, the pressure coils of the wattmeter and the voltmeter are connected such that the current in them does not pass through the current coil of the wattmeter.

Sometimes, a high-resistance voltmeter is connected across the secondary. The reading of the voltmeter gives the induced e.m.f. in the secondary winding. This helps to find transformation ratio K.

The no-load vector diagram is shown in Figure (13). If W is the wattmeter reading [in Figure (34)], then

$$W = V_1 I_0 \cos \varphi_0$$

$$\cos \varphi_0 = W/V_1 I_0$$

$$\therefore \quad I_\mu = I_0 \sin \varphi_0,$$

$$I_w = I_0 \cos \varphi_0$$

$$\therefore \quad \boxed{X_0 = V_1/I_\mu}$$

and

$$\boxed{R_0 = V_1/I_w}$$

Or since the current is practically all-exciting current when a transformer is on no-load (i.e. $I_0 \cong I_\mu$) and as the voltage drop in primary leakage impedance is small (If it is not negligibly small, then $I_0 = E_1 Y_0$ i.e. instead of V_1 we will have to use E_1), hence the exciting admittance Y_0 of the transformer is given by,

$$I_0 = V_1 Y_0$$

or $$\boxed{Y_0 = I_0/V_1}$$

The exciting conductance G_0 is given by,

$$W = V_1^2 G_0$$

or

$$\boxed{G_0 = W/V_1^2}$$

The exciting susceptance, $\boxed{B_0 = \sqrt{Y_0^2 - G_0^2}}$

Example 21: In no-load test of single-phase transformer, the following test data were obtained :
Primary voltage : 220 V ; Secondary voltage : 110 V ;
Primary current : 0.5 A ; Power input : 30 W.
Find the following :
(i) The turns ratio **(ii)** the magnetising component of no-load current **(iii)** its working (or loss) component **(iv)** the iron loss.
Resistance of the primary winding = 0.6 ohm.

Solution:

(i) Turn ratio, $N_1/N_2 = 220/110 = 2$

(ii) $W = V_1 I_0 \cos \varphi_0$

$\cos \varphi_0 = 30/220 \times 0.5 = 0.273$

$\sin \varphi_0 = 0.962$

$I_\mu = I_0 \sin \varphi_0 = 0.5 \times 0.962 = 0.48$ A

(iii) $I_w = I_0 \cos \varphi_0 = 0.5 \times 0.273 = 0.1365$ A

(iv) Primary Cu loss $= I_0^2 R_1 = 0.5^2 \times 0.6 = 0.15$ W

$\therefore$ Iron loss $= 30 - 0.15 = 29.85$ W

Separation of Core Losses

The core loss of a transformer depends upon the frequency and the maximum flux density when the volume and the thickness of the core laminations are given. The core loss is made up of two parts,

(i) hysteresis loss $W_h = PB_{max}^{1.6} f$ as given by Steinmetz's empirical relation

Where, $W_h = \eta B_{max}^{1.6} f V$

As, (ηV) is constant and equal to a constant (P).

$\therefore W_h = PB_{max}^{1.6} f$

(ii) eddy current loss $W_e = QB_{max}^{2} f^{2}$ where Q is a constant.

Where, $W_e = KB_{max}^{2} f^{2} t^2 V^2$

As, ($K t^2 V^2$) is constant and equal to a constant (Q).

$\therefore W_e = QB_{max}^{2} f^{2}$

The total core-loss is given by

$W_i = W_h + W_e = PB_{max}^{1.6} f + Q B_{max}^{2} f^{2}$

If we carry out two experiments using two different frequencies but the same maximum flux density, we should be able to find the constants P and Q and hence calculate hysteresis and eddy current losses separately.

Example 22: In a transformer, the core loss is found to be 52 W at 40 Hz and 90 W at 60 Hz measured at same peak flux density. Compute the hysteresis and eddy current losses at 50 Hz.

Solution:

Note: the flux density is constant because the iron losses are measured at same peak flux density for both cases.

Since the flux density is the same in both cases, we can use the relation

Total core loss $W_i = Af + Bf^2$

Or $W_i/f = A + Bf$

$\therefore$ $52/40 = A + 40B$ and $90/60 = A + 60B$

$\therefore$ $A = 0.9$ and $B = 0.01$

At 50 Hz, the two losses are

$W_h = Af = 0.9 \times 50 = 45$ W ; $W_e = Bf^2 = 0.01 \times 50^2 = 25$ W

Example 23: In a power loss test on a 10 kg specimen of sheet steel laminations, the maximum flux density and waveform factor are maintained constant and the following results were obtained:

Frequency (Hz)	25	40	50	60	80
Total loss (watt)	18.5	36	50	66	104

Calculate the eddy current loss per kg at a frequency of 50 Hz.

Solution:

When flux density and wave form factor remain constant, the expression for iron loss can be written as

$$W_i = Af + Bf^2 \qquad \text{or} \qquad W_i/f = A + Bf$$

The values of (W_i/f) for different frequencies are as under :

f	25	40	50	60	80
W_i/f	0.74	0.9	1.0	1.1	1.3

The graph between f and (W_i/f) has been plotted in Figure (35). As seen from it, A = 0.5 and B = 0.01

$\therefore$ Eddy current loss at 50 Hz $= Bf^2 = 0.01 \times 50^2 = 25$ W

$\therefore$ Eddy current loss/kg $= 25/10 = 2.5$ W

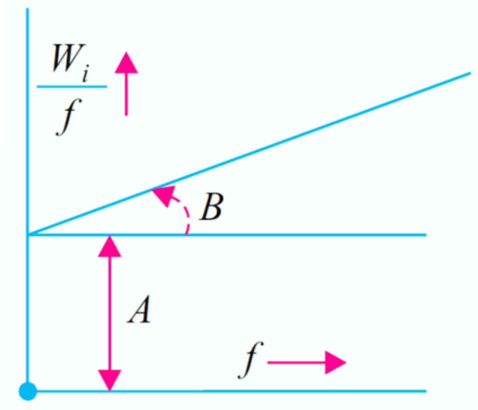


Figure (35)

Example 24: In a test for the determination of the losses of a 440-V, 50-Hz transformer, the total iron losses were found to be 2500 W at normal voltage and frequency. When the applied voltage and frequency were 220 V and 25 Hz, the iron losses were found to be 850 W. Calculate the eddy-current loss at normal voltage and frequency.

Solution:

Note: if the applied voltage and frequency are increased or decreased in the same proportion, this means flux density is constant because,

$$E = 4.44 f \Phi_{max} N = 4.44 f N B_{max} A, \text{ as } (4.44 N A = \text{constant})$$

$$\therefore \quad E \propto B_{max} f$$

Or $$B_{max} \propto \frac{E}{f}$$

The flux density in both cases is the same because in second case voltage as well as frequency are halved. Flux density remaining the same, the eddy current loss is proportional to f^2 and hysteresis loss $\propto$ f.

$$W_h \propto f \Longrightarrow W_h = A f$$

and $$W_e \propto f^2 \Longrightarrow W_e = B f^2$$

where A and B are constants.

Total iron loss $W_i = Af + Bf^2$

$$\therefore \quad \frac{W_i}{f} = A + Bf \qquad \text{...(i)}$$

Now, when f = 50 Hz : W_i = 2500 W

and when f = 25 Hz ; W_i = 850 W

Using these values in (i) above, we get

$$2{,}500/50 = A + 50\,B$$

And $$850/25 = A + 25\,B$$

$$\therefore \quad B = 16/25 = 0.64$$

Hence, at normal p.d. and frequency

$$W_e = Bf^2 = 0.64 \times 50^2 = \mathbf{1600\ W}$$

$$W_h = 2500 - 1600 = 900\ W$$

Example 25: When a transformer is connected to a 1000-V, 50-Hz supply the core loss is 1000 W, of which 650 is hysteresis and 350 is eddy current loss. If the applied voltage is raised to 2,000 V and the frequency to 100 Hz, find the new core losses.

Solution:

Hysteresis loss $W_h \propto B_{max}^{1.6} f = PB_{max}^{1.6} f$

Eddy current loss $W_e \propto B_{max}^{2} f^{2} = QB_{max}^{2} f^{2}$

From the relation $E = 4.44\, fN_{Bmax} A$ volt, we get $B_{max} \propto E/f$

Putting this value of B_{max} in the above equations, we have

$$\boxed{W_h = P\left(\frac{E}{f}\right)^{1.6} \times f = P\,E^{1.6} f^{-0.6}}$$

and

$$\boxed{W_e = Q\left(\frac{E}{f}\right)^{2} \times f^2 = Q\,E^2}$$

In the first case,

$E = 1000$ V, $f = 50$ Hz, $W_h = 650$ W, $W_e = 350$ W

$\therefore \quad 650 = P \times 1000^{1.6} \times 50^{-0.6}$

$\therefore \quad P = 650 \times 1000^{-1.6} \times 50^{0.6}$

Similarly, $350 = Q \times 1000^2$

$\therefore \quad Q = 350 \times 1000^{-2}$

Hence, constants P and Q are known.

Using them in the second case, we get

$W_h = (650 \times 1000^{-1.6} \times 50^{0.6}) \times 2000^{1.6} \times 100^{-0.6} = 650 \times 2 = 1{,}300$ W

$W_e = (350 \times 1000^{-2}) \times 2{,}000^2 = 350 \times 4 = 1{,}400$ W

$\therefore$ Core loss under new condition is = 1,300 + 1,400 = **2700 W**

Alternative Solution

Here, both voltage and frequency are doubled, leaving the flux density unchanged.

With 1000 V at 50 Hz

$W_h = Af$ or $650 = 50\,A$; $A = 13$

$W_e = Bf^{2}$ or $350 = B \times 50^2$; $B = 7/50$

With 2000 V at 100 Hz

$W_h = A\,f = 13 \times 100 = 1300$ W and

$W_e = Bf^{2} = (7/50) \times 100^2 = 1400$ W

$\therefore$ New core loss = 1300 + 1400 = **2700 W**

Example 26: A transformer with normal voltage impressed has a flux density of 1.4 Wb/m^2 and a core loss comprising of 1000 W eddy current loss and 3000 W hysteresis loss. What do these losses become under the following conditions ?
(a) increasing the applied voltage by 10% at rated frequency.
(b) reducing the frequency by 10% with normal voltage impressed.
(c) increasing both impressed voltage and frequency by 10 per cent.

Solution:

As seen from Example 25

$W_h = PE^{1.6} f^{-0.6}$ and $W_e = QE^2$

From the given data, we have $3000 = PE^{1.6} f^{-0.6}$ ...**(i)**

and $1000 = QE^2$ ...**(ii)**

where E and f are the normal values of primary voltage and frequency.

(a) Here voltage becomes = E + 10% E = 1.1 E

The new hysteresis loss is $W_h = P\,(1.1\,E)^{1.6} f^{-0.6}$ ...**(iii)**

Dividing Eq. **(iii)** by **(i)**, we get

$$\frac{W_h}{3000} = 1.1^{1.6}$$

∴ $W_h = 3000 \times 1.165 =$ **3495 W**

The new eddy-current loss is

$$W_e = Q\,(1.1.\,E)^2$$

∴ $$\frac{W_e}{1000} = 1.1^2$$

∴ $W_e = 1000 \times 1.21 =$ **1210 W**

(b) As seen from Eq. **(i)** above eddy-current loss would not be effected. The new hysteresis loss is,

$W_h = PE^{1.6} (0.9\,f)^{-0.6}$ ...**(iv)**

From **(i)** and **(iv)**, we get, $\frac{W_h}{3000} = 0.9^{1.6}$

$W_h = 3000 \times 1.065 =$ **3,196 W**

(c) In this case, both E and f are increased by 10%. The new losses are as under :

$$W_h = P\,(1.1\,E)^{1.6}\,(1.1\,f)^{-0.6}$$

∴ $$\frac{W_h}{3000} = 1.1^{1.6} \times 1.1^{-0.6} = 1.165 \times 0.944$$

∴ $W_h = 3000 \times 1.165 \times 0.944 =$ **3,299 W**

As We is unaffected by changes in f, its value is the same as found in (a) above i.e. **1210 W**

Example 27: A transformer is connected to 2200 V, 40 Hz supply. The core-loss is 800 watts out of which 600 watts are due to hysteresis and the remaining, eddy current losses. Determine the core-loss if the supply voltage and frequency are 3300 V and 60 Hz respectively.

Solution:

Note: B_{max} is constant because (E/f = 2200/40 = 3300/60 = 55 ,same for both cases).

For constant flux density (i.e. constant V/f ratio), which is fulfilled by 2200/40 or 3300/60 figures in two cases,

Core-loss = $A\,f + B\,f^2$

First term on the right-hand side represents hysteresis-loss and the second term represents the eddy-current loss.

At 40 Hz, 800 = 600 + eddy current loss.

Thus, $A\,f = 600$, or $A = 15$

$B\,f^2 = 200$, or $B = 200/1600 = 0.125$

At 60 Hz, core-loss = $15 \times 60 + 0.125 \times 60^2 = 900 + 450 = 1350$ watts

(ii)Short-Circuit or Impedance Test

This is an economical method for determining the following:

(i) Equivalent impedance ($\mathbf{Z}_{01}$ or $\mathbf{Z}_{02}$), leakage reactance (X_{01} or X_{02}) and total resistance (R_{01} or R_{02}) of the transformer as referred to the winding in which the measuring instruments are placed.

(ii) Cu loss at full load (and at any desired load). This loss is used in calculating the efficiency of the transformer.

(iii) Knowing Z_{01} or Z_{02}, the total voltage drop in the transformer as referred to primary or secondary can be calculated and hence regulation of the transformer determined.

In this test, one winding, usually the low-voltage winding, is solidly short-circuited by a thick conductor (or through an ammeter which may serve the additional purpose of indicating rated load current) as shown in Figure (36).

A low voltage (usually 5 to 10% of normal primary voltage) at correct frequency (though for Cu losses it is not essential) is applied to the primary and is cautiously increased till full-load currents are flowing both in primary and secondary (as indicated by the respective ammeters).

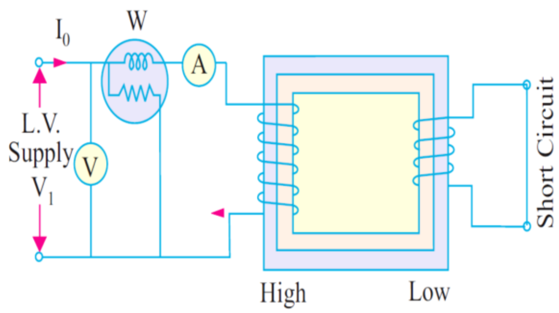


Figure (36)

Since, in this test, the applied voltage is a small percentage of the normal voltage, the mutual flux Φ produced is also a small percentage of its normal value. Hence, core losses are very small with the result that the wattmeter reading represent the full-load Cu loss or I^2R loss for the whole transformer *i.e.* both primary Cu loss and secondary Cu loss. The equivalent circuit of the transformer under short-circuit condition is shown in Figure (37).

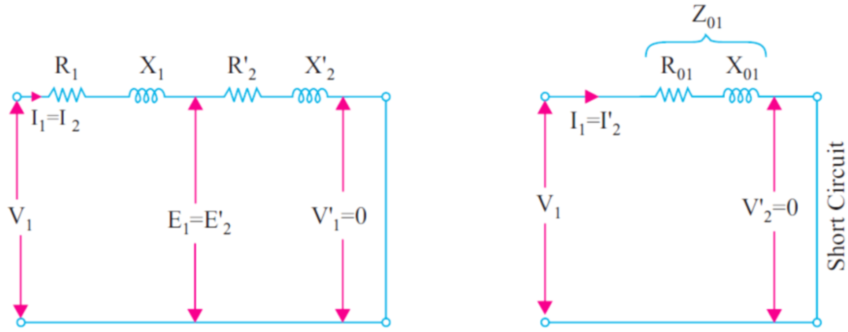


Figure (37)

If V_{sc} is the voltage required to circulate rated load currents, then $Z_{01} = V_{sc}/I_1$

Also $W = I_1^2\, R_{01}$

$$\therefore\quad R_{01} = W/I_1$$

$$\therefore\quad X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

In Figure (38-a) the equivalent circuit vector diagram for the short-circuit test is shown.
It is obvious that the entire voltage V_{SC} is consumed in the impedance drop of the two windings.
If R_1 can be measured, then knowing R_{01}, we can find $R_2' = R_{01} - R_1$. The impedance triangle can then be divided into the appropriate equivalent triangles for primary and secondary as shown in Figure (38-b).

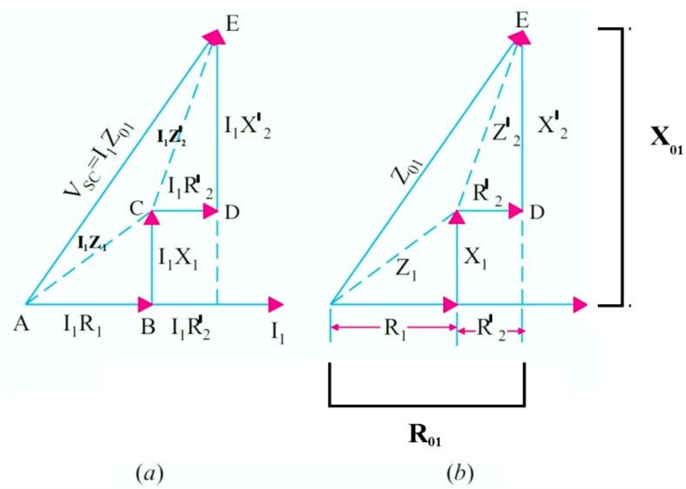


Figure (38)

Why Transformer Rating in kVA ?

As seen, Cu loss of a transformer depends on current and iron loss on voltage. Hence, total transformer loss depends on volt-ampere (VA) and not on phase angle between voltage and current *i.e.* it is independent of load power factor. That is why rating of transformers is in KVA and not in kW.

Example 28: The primary and secondary windings of a 30 kVA 6000/230, V, 1-phase transformer have resistance of 10 ohm and 0.016 ohm respectively. The reactance of the transformer referred to the primary is 34 ohm. Calculate the primary voltage required to circulate full-load current when the secondary is short-circuited. What is the power factor on short circuit ?

Solution:

$K = 230/6000 = 23/600$, $X_{01} = 34\ \Omega$,

$R_{01} = R_1 + R_2/K^2 = 10 + 0.016\ (600/23)^2 = 20.9\ \Omega$

$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{20.9^2 + 34^2} = 40\ \Omega$

F.L., $I_1 = 30{,}000/6000 = 5$ A

$V_{SC} = I_1 Z_{01} = 5 \times 40 = 200$ V

Short circuit p.f. $= R_{01}/Z_{01} = 20.9/40 = 0.52$

Example 29: Obtain the equivalent circuit of a 200/400-V, 50-Hz, 1-phase transformer from the following test data :

O.C test : 200 V, 0.7 A, 70 W – on L.V. side

S.C. test : 15 V, 10 A, 85 W – on H.V. side

Calculate the secondary voltage when delivering 5 kW at 0.8 p.f. lagging, the primary voltage being 200V.

Solution:

From O.C. Test

$V_1 I_0 \cos\varphi_0 = W_0$

$\therefore\quad 200 \times 0.7 \times \cos\varphi_0 = 70$

$\cos\varphi_0 = 0.5$ and $\sin\varphi_0 = 0.866$

$I_w = I_0 \cos\varphi_0 = 0.7 \times 0.5 = 0.35$ A

$I_\mu = I_0 \sin\varphi_0 = 0.7 \times 0.866 = 0.606$ A

$R_0 = V_1/I_w = 200/0.35 = 571.4\ \Omega$

$X_0 = V_1/I_\mu = 200/0.606 = 330\ \Omega$

As shown in Figure (39), these values refer to primary i.e. low-voltage side.

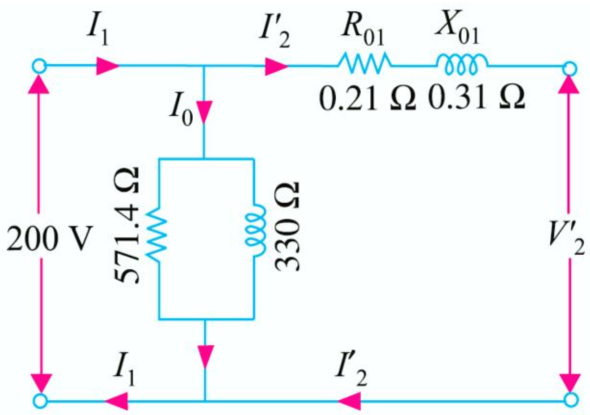


Figure (39): approximate model

From S.C. Test

It may be noted that in this test, instruments have been placed in the secondary i.e. high voltage winding whereas the low-voltage winding i.e. primary has been short-circuited.

$Z_{02} = V_{sc}/I_2 = 15/10 = 1.5\ \Omega$; $K = 400/200 = 2$

$Z_{01} = Z_{02}/K^2 = 1.5/4 = 0.375\ \Omega$

Also $I_2^2 R_{02} = W$; $R_{02} = 85/100 = 0.85\ \Omega$

$R_{01} = R_{02}/K^2 = 0.85/4 = 0.21\ \Omega$

$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{0.375^2 - 0.21^2} = 0.31\ \Omega$

Output kVA = 5/0.8 ; Output current $I_2 = 5000/(0.8 \times 400) = 15.6$ A

This value of I_2 is approximate because V_2 (which is to be calculated as yet) has been taken equal to 400 V (which, in fact, is equal to E_2 or ${}_0V_2$).

Now, $Z_{02} = 1.5\ \Omega$, $R_{02} = 0.85\ \Omega$ ∴ $X_{02} = \sqrt{1.5^2 - 0.85^2} = 1.24\ \Omega$

Total transformer drop as referred to secondary $= I_2 (R_{02} \cos \varphi_2 + X_{02} \sin \varphi_2)$

$= 15.6\,(0.85 \times 0.8 + 1.24 \times 0.6) = 22.2$ V

∴ $V_2 = 400 - 22.2 =$ **377.8 V**

Example 30: Starting from the ideal transformer, obtain the approximate equivalent circuit of a commercial transformer in which all the constants are lumped and represented on one side. A 1-phase transformer has a turn ratio of 6. The resistance and reactance of primary winding are 0.9 Ω and 5 Ω respectively and those of the secondary are 0.03 Ω and 0.13 Ω respectively. If 330 -V at 50-Hz be applied to the high voltage winding with the low-voltage winding short-circuited, find the current in the low-voltage winding and its power factor. Neglect magnetizing current.

Solution:

Here K = 1/6

$R_{01} = R_1 + R'_2 = 0.9 + (0.03 \times 36) = 1.98\ \Omega$

$X_{01} = X_1 + X_2' = 5 + (0.13 \times 36) = 9.68\ \Omega$

∴ $Z_{01} = \sqrt{9.68^2 + 1.98^2} = 9.9\ \Omega$

$V_{SC} = 330$ V

∴ Full-load primary current $I_1 = V_{sc}/Z_{01} = 330/0.9 = 100/3$ A

As I_0 is negligible, hence $I_1 = I'_2 = 100/3$ A.

Now, $I'_2 = KI_2$

F.L. secondary current $I_2 = I'_2/K = (100/3) \times 6 = 200$ A

Now, Power input on short-circuit $= V_{SC} I_1 \cos \varphi_{SC} =$ Cu loss $= I_1^2 R_{01}$

∴ $(100/3)^2 \times 1.98 = 330 \times (100/3) \times \cos \varphi_{SC}$

∴ $\cos \varphi_{SC} =$ **0.2**

Or;

$$\cos \varphi_{SC} = \frac{R_{02}}{Z_{02}} = \frac{R_{01}}{Z_{01}} = \frac{1.98}{9.9} = \mathbf{0.2}$$

Example 31: A 1-phase, 10-kVA, 500/250-V, 50-Hz transformer has the following constants:
Reactance : primary 0.2 Ω; secondary 0.5 Ω
Resistance : primary 0.4 Ω; secondary 0.1 Ω
Resistance of equivalent exciting circuit referred to primary, $R_0 = 1500\ \Omega$
Reactance of equivalent exciting circuit referred to primary, $X_0 = 750\ \Omega$
What would be the reading of the instruments when the transformer is connected for the open circuit and short-circuit tests ?

Solution:

O.C. Test

$I_\mu = V_1/X_0 = 500/750 = 2/3$ A ; $I_w = V_1/R_0 = 500/1500 = 1/3$ A

$\therefore\ I_0 = \sqrt{[(1/3)^2 + (2/3)^2]} = 0.745$ A

No-load primary input $= V_1 I_w = 500 \times 1/3 = 167$ W

Instruments used in primary circuit are : voltmeter, ammeter and wattmeter, their readings being 500 V, 0.745 A and 167 W respectively.

S.C. Test

Suppose S.C. test is performed by short-circuiting the l.v. winding i.e. the secondary so that all instruments are in primary.

$R_{01} = R_1 + R'_2 = R_1 + R_2/K^2$; Here $K = 1/2$ $\Rightarrow$ $\therefore\ R_{01} = 0.2 + (4 \times 0.5) = 2.2\ \Omega$

Similarly, $X_{01} = X_1 + X'_2 = 0.4 + (4 \times 0.1) = 0.8\ \Omega$

$Z_{01} = \sqrt{[(2.2)^2 + (0.8)^2]} = 2.341\ \Omega$

Full-load primary current

$I_1 = 10{,}000/500 = 20$ A $\therefore V_{SC} = I_1 Z_{01} = 20 \times 2.341 = 46.8$ V

Power absorbed $= I_1^2 R_{01} = 20^2 \times 2.2 = 880$ W

Primary instruments will read : **46.8 V, 20 A, 880 W.**

Example 32: The efficiency of a 1000-kVA, 110/220 V, 50-Hz, single-phase transformer, is 98.5 % at half full-load at 0.8 p.f. leading and 98.8 % at full-load unity p.f. Determine (i) iron loss (ii) full-load copper loss and (iii) maximum efficiency at unity p.f.

Solution:

Note:

- Half Full Load (H.FL.) means half output power (active, apparent, and reactive) also half output current of Full Load (F.L.).
- F.L. Cu loss $= (I_{F.L.})^2 R$, $\qquad I_{H.F.L.} = \frac{1}{2} I_{F.L}$

  H.F.L. Cu loss $= (I_{H.F.L.})^2 R = (\frac{1}{2} I_{F.L.})^2 R = \frac{1}{4} (I_{F.L.})^2 R = \frac{1}{4}$ (F.L. Cu loss)
- $S_{F.L.} = V.I_{F.L.}$

  $S_{H.F.L.} = V.I_{H.F.L.} = V(\frac{1}{2} I_{F.L}) = \frac{1}{2} V.I_{F.L.} = \frac{1}{2} S_{F.L.}$

At F.L.

Output at F.L. unity p.f. ($P_{out\ at\ F.L.}$) = $S_{F.L.}\cos\varphi$ = 1000 × 1 = 1000 kW
F.L. input ($P_{input\ at\ F.L.}$) = ($P_{out\ at\ F.L.}$)/efficiency = 1000/0.988 = 1012.146 kW
F.L. lossses = 1012.146 − 1000 = 12.146 kW
If F.L. Cu and iron losses are x and y respectively then, [where x= F.L. Cu loss = $(I_{F.L.})^2R$]
x + y = 12.146 Kw ...**(*i*)**

At H.F.L.

$S_{H.F.L.} = \frac{1}{2} S_{F.L.} = \frac{1}{2} \times 1000 = 500$ KVA
Output at H.F.L., 0.8 p.f. ($P_{out\ at\ H.F.L.}$) = $S_{H.F.L.}\cos\varphi$ = 500 × 0.8 = 400 kW
H.F.L. input ($P_{input\ at\ H.F.L.}$) = ($P_{out\ at\ H.F.L.}$)/efficiency = 400/0.985 = 406.091 kW
Total losses at half F.L. = 406.091 − 400 = 6.091 kW

Cu loss at half-load = x $(1/2)^2$ = x/4, [H.F.L. Cu loss = $\frac{1}{4}$ (F.L. Cu loss)]

∴ x/4 + y = 6.091 ...**(ii)**

From Eqn. **(i)** and **(ii)**, we get **(i)** x = 8.073 kW and (ii) y = 4.073 Kw

(iii) At max. efficiency,

Cu loss = Iron loss

$\therefore (I_{at\ max.\ efficiency})^2 R = W_i$

$I_{at\ max.\ efficiency} = \sqrt{\frac{W_i}{R}}$, multiply both side by (V) & right side by ($I_{F.L.}/I_{F.L.}$)

$(I_{at\ max.\ efficiency}) \times V = V \times (I_{F.L.}/I_{F.L.}) \times \sqrt{\frac{W_i}{R}}$

$= V . I_{F.L.} \times \sqrt{\frac{W_i}{I^2_{F.L.} R}}$

$S_{max} = S_{F.L.} \times \sqrt{\frac{W_i}{\text{F.L.Cu loss}}}$

kVA for η_{max} = 1000 × $\sqrt{4.073/8.073}$ = 710.3 kVA
Output at u.p.f. = 710.3 × 1 = 710.3 kW
Cu loss = iron loss = 4.037 kW ; Total loss = 2 × 4.037 = 8.074 kW

∴ η_{max} = 710.3/(710.3 + 8.074) = 0.989 or 98.9 %

Example 33: The equivalent circuit for a 200/400-V step-up transformer has the following

parameters referred to the low-voltage side.
Equivalent resistance = 0.15 Ω ; Equivalent reactance = 0.37 Ω
Core-loss component resistance = 600 Ω ; Magnetising reactance = 300 Ω
When the transformer is supplying a load at 10 A at a power factor of 0.8 lag, calculate (i) the primary current (ii) secondary terminal voltage.

Solution:

We are given the following :

$R_{01} = 0.15\ \Omega$, $X_{01} = 0.37\ \Omega$; $R_0 = 600\ \Omega$, $X_0 = 300\ \Omega$

Using the approximate equivalent circuit of Figure (40), we have,

$I_\mu = V_1/X_0 = 200/300 = (2/3)$ A

$I_w = V_1/R_0 = 200/600 = (1/3)$ A

$I_0 = \sqrt{I_\mu^2 + I_w^2} = \sqrt{(2/3)^2 + (1/3)^2} = 0.745$ A

As seen from Figure (40)

$\tan\theta = I_w / I_\mu = (1/3) / (2/3) = 1/2$

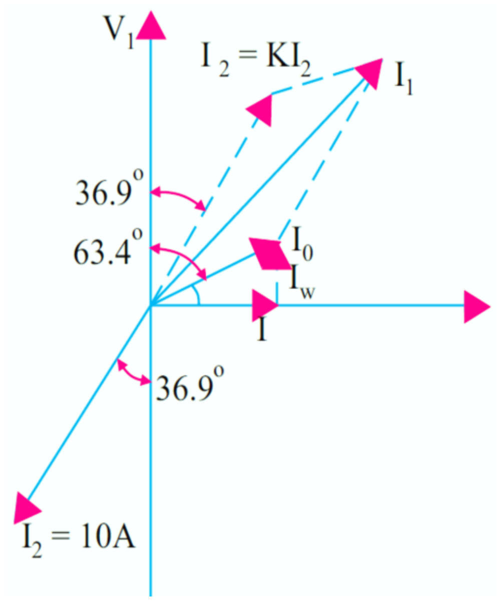
$\therefore \theta = 26.6^o$

$\therefore \quad \varphi_0 = 90^o - 26.6^o = 63.4^o$; Angle between I_0 and

Or $\quad \varphi_0 = \tan^{-1}(I_\mu / I_w) = 63.4^o$

$I'_2 = 63.4^o - 36.9^o = 26.5^o$; $K = 400/200 = 2$

$I'_2 = KI_2 = 2 \times 10 = 20$ A



(i) $I_1 = \sqrt{I_0^2 + I_2'^2 + 2 \times I_0 \times I_2' \times cos\varphi_{between\ them}}$

$= (0.745^2 + 20^2 + 2 \times 0.745 \times 20 \times \cos 26.5^o)^{1/2} =$ **20.67 A**

Figure (40)

(ii) $R_{02} = K^2 R_{01} = 2^2 \times 0.15 = 0.6\ \Omega$

$X_{02} = 22 \times 0.37 = 1.48\ \Omega$

Approximate voltage drop $= I_2 (R_{02} \cos\varphi + X_{02} \sin\varphi)$

$= 10\,(0.6 \times 0.8 + 1.48 \times 0.6) = 13.7$ V

$\therefore$ Secondary terminal voltage = 400 − 13.7 = **386.3 V**

Example 34: The low voltage winding of a 300-kVA, 11,000/2500-V, 50-Hz transformer has 190 turns and a resistance of 0.06. The high-voltage winding has 910 turns and a resistance of 1.6 Ω . When the l.v. winding is short-circuited, the full-load current is obtained with 550-V applied to the h.v. winding. Assume full-load efficiency is 0.985. Calculate (i) the equivalent resistance and leakage reactance as referred to h.v. side and (ii) the leakage reactance of each winding. If you know that for each winding the ratio (reactance/resistance) is the same

Solution:

the full-load primary current is = 300,000/0.985 × 11,000 = 27.7A

∴ $Z_{01} = 550/27.7 = 19.8\ \Omega$; $R'_2 = R_2/K^2 = 0.06\ (910/190)^2 = 1.38\ \Omega$

$R_{01} = R_1 + R'_2 = 1.6 + 1.38 = 2.98\ \Omega$

$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{19.8^2 - 2.98^2} = 19.5\ \Omega$

(a) $R_{01} =$ **$2.98\ \Omega$** ; $X_{01} =$ **$19.5\ \Omega$**

(b) As the ratio of (reactance/resistance) is the same then,

$X_1 = 19.5 \times 1.6/2.98 = 10.5\ \Omega$

$X'_2 = 19.5 \times 1.38/2.98 = 9.0\ \Omega$; $X_2 = 9(190/910)2 = 0.39\ \Omega$

$X_1 =$ **$10.5\ \Omega$** : $X_2 =$ **$0.39\ \Omega$**

Example 35: A 230/115 volts, single phase transformer is supplying a load of 5 Amps, at power factor 0.866 lagging. The no-load current is 0.2 Amps at power factor 0.208 lagging. Calculate the primary current and primary power factor

Solution:

L.V. current of 5 amp is referred to as a 2.5 amp current on the primary (= H.V.) side, at 0.866 lagging p.f. To this, the no load current should be added, as per the phasor diagram in Figure (41). The phase angle of the load-current is 30° lagging. The no load current has a phase angle of 80° lagging. Resultant of these two currents has to be worked out. Along the reference, active components are added.

Active components of currents = 2.5 × 0.866 + 0.2 × 0.208 = 2.165 + 0.0416 = 2.2066 A

Along the perpendicular direction, the reactive components get added up.

Reactive component = 2.5 × 0.5 + 0.2 × 0.9848 = 1.25 + 0.197 = 1.447 A.

$I_1 = 2.2066 - j\ 1.447$

$\varphi = \tan^{-1}(1.447 / 2.2066) =$ **33.25° ; as shown**

Or: (using complex number methods)

$I_2 = 5\ \angle 30°\ A$

$I_0 = 0.2\ \angle 78°\ A$

$I'_2 = KI_2 = (115/230) \times 5\ \angle 30° = 2.5\ \angle 30°\ A$

$I_1 = I_0 + I'_2 = (0.2\ \angle 78°) + (2.5\ \angle 30°)$

$= 2.6\ \angle 33.17°\ A$

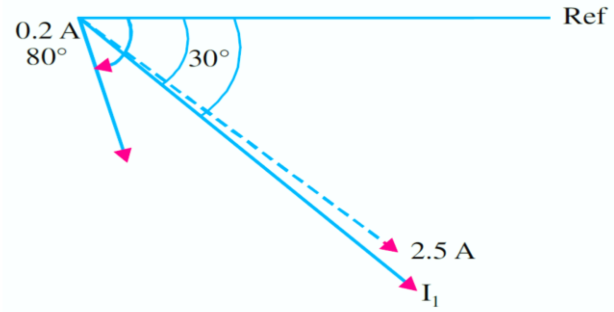


Figure (41): Phasor diagram for currents

Tutorial Problems (3)

[1] The S.C. test on a 1-phase transformer, with the primary winding short-circuited and 30 V applied to the secondary gave a wattmeter reading of 60 W and secondary current of 10 A. If the normal applied primary voltage is 200, the transformation ratio 1 :2 and the full-load secondary current 10 A, calculate the secondary terminal p.d. at full-load current for (a) unity power factor (b) power factor 0.8 lagging. If any approximations are made, they must be explained. **[394 V, 377.6 V]**

[2] A single-phase transformer has a turn ratio of 6, the resistances of the primary and secondary windings are 0.9 Ω and 0.025 Ω respectively and the leakage reactances of these windings are 5.4 Ω and 0.15 Ω respectively. Determine the voltage to be applied to the low-voltage winding to obtain a current of 100 A in the short-circuited high voltage winding. Ignore the magnetising current. **[82 V]**

[3] Draw the equivalent circuit for a 3000/400-V, I-phase transformer on which the following test results were obtained. Input to high voltage winding when 1.v. winding is open-circuited : 3000 V, 0.5 A, 500 W. Input to 1.v. winding when h.v. winding is short-circuited : 11 V, 100 A, 500 W. Insert the appropriate values of resistance and reactance.
[R_0 = 18,000 Ω, X_0 = 6,360 Ω, R_{01} = 2.81 Ω, X_{01} = 5.51 Ω]

[4] The iron loss in a transformer core at normal flux density was measured at frequencies of 30 and 50 Hz, the results being 30 W and 54 W respectively. Calculate (a) the hysteresis loss and (b) the eddy current loss at 50 Hz. **[44 W, 10 W]**

[5] An iron core was magnetised by passing an alternating current through a winding on it. The power required for a certain value of maximum flux density was measured at a number of different frequencies. Neglecting the effect of resistance of the winding, the power required per kg of iron was 0.8 W at 25 Hz and 2.04 W at 60 Hz. Estimate the power needed per kg when the iron is subject to the same maximum flux density but the frequency is 100 Hz. **[3.63 W]**

[6] The ratio of turns of a 1-phase transformer is 8, the resistances of the primary and secondary windings are 0.85 Ω and 0.012 Ω respectively and leakage reactances of these windings are 4.8 Ω and 0.07 Ω respectively. Determine the voltage to be applied to the primary to obtain a current of 150 A in the secondary circuit when the secondary terminals are short-circuited. Ignore the magnetizing current. **[176.4 W]**

[7] A transformer has no-load losses of 55 W with a primary voltage of 250 V at 50 Hz and 41 W with a primary voltage of 200 V at 40 Hz. Compute the hysteresis and eddy current losses at a primary voltage of 300 volts at 60 Hz of the above transformer. Neglect small amount of copper loss at no load. **[43.5 W ; 27 W]**

[8] A 20 kVA, 500/250 V, 50 Hz, 1-phase transformer has the following test results :
O.C. Test (1.v. side) : 250 V, 1.4 A, 105 W
S.C. Test (h.v. side) : 104 V, 8 A, 320 W
Compute the parameters of the approximate equivalent circuit referred to the low voltage side and draw the circuit. **[R_0 = 592.5 Ω ; X_0 = 187.2 Ω ; R_{02} = 1.25 Ω ; X_{12} = 3 Ω]**

[9] A 10-kVA, 2000/400-V, single-phase transformer has resistances and leakage reactances as follows :
$R_1 = 5.2\ \Omega$, $X_1 = 12.5\ \Omega$, $R_2 = 0.2\ \Omega$, $X_2 = 0.5\ \Omega$
Determine the value of secondary terminal voltage when the transformer is operating with rated primary voltage with the secondary current at its rated value with power factor 0.8 lag. The no-load current can be neglected. Draw the phasor diagram. **[376.8 V]**

[10] A 1000-V, 50-Hz supply to a transformer results in 650 W hysteresis loss and 400 W eddy current loss. If both the applied voltage and frequency are doubled, find the new core losses. **[W_h = 1300 W ; W_e = 1600 W]**

[11] A 50 kVA, 2200/110 V transformer when tested gave the following results :
O.C. test (L.V. side) : 400 W, 10 A, 110 V.
S.C. test (H.V. side) : 808 W, 20.5 A, 90 V.
Compute all the parameters of the equivalent circuit. referred to the H.V. side and draw the resultant circuit. **[Shunt branch : R_0 = 12.1 k-ohms, X_m = 4.724 k-ohms Series branch : r = 1.923 ohms, x = 4.39 ohms]**

18. Regulation of a Transformer

(1) When a transformer is loaded with a **constant primary voltage**, the secondary voltage decreases (Assuming lagging power factor. It will increase if power factor is leading) because of its internal resistance and leakage reactance.

Let $_0V_2$ = secondary terminal voltage at no-load.
$= E_2 = EK_1 = KV_1$ because at no-load the impedance drop is negligible.
V_2 = secondary terminal voltage on full-load.

The change in secondary terminal voltage from no-load to full-load is $= {_0V_2} - V_2$. This change divided by $_0V_2$ is known as regulation 'down'. If this change is divided by V_2, i.e., full-load secondary terminal voltage, then it is called regulation 'up'.

$$\therefore \quad \%\,\text{regn 'down'} = \frac{_0V_2 - V_2}{_0V_2} \times 100 \quad \text{and} \quad \%\,\text{regn 'up'} = \frac{_0V_2 - V_2}{V_2} \times 100$$

In further treatment, unless stated otherwise, regulation is to be taken as regulation 'down'. We have already seen that the change in secondary terminal voltage from no load to full-load, expressed as a percentage of no-load secondary voltage is,

$$\boxed{= v_r \cos\varphi \pm v_x \sin\varphi \qquad \text{(approximately)}}$$

Or more accurately

$$\boxed{= (v_r \cos\varphi \pm v_x \sin\varphi) + \frac{1}{200}(v_x \cos\varphi \pm v_r \sin\varphi)^2}$$

$$\therefore \quad \boxed{\%\,\text{regn} = v_r \cos\varphi \pm v_x \sin\varphi} \qquad \text{...approximately.}$$

The lesser this value, the better the transformer, because a good transformer should keep its secondary terminal voltage as constant as possible under all conditions of load.

(2) The regulation may also be explained in terms of primary values.
In Figure (42)(a) the approximate equivalent circuit of a transformer is shown and in Figure (42) (b), (c) and (d) the vector diagrams corresponding to different power factors are shown. The secondary **no-load** terminal voltage as referred to primary is $E'_2 = E_2/K = E_1 = V_1$ and if the secondary full-load voltage as referred to primary is V'_2 ($= V_2/K$) then,

$$\%\,\text{regn} = \frac{V_1 - V'_2}{V_1} \times 100$$

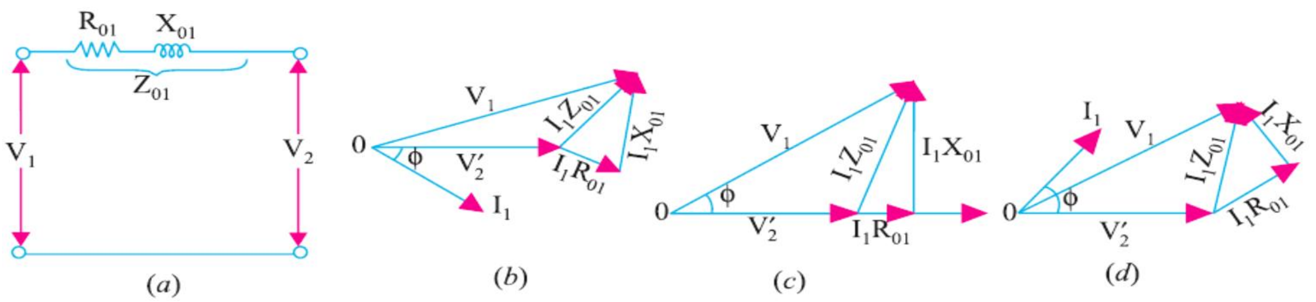


Figure (42)

From the vector diagram, it is clear that if angle between V_1 and V'_2 is neglected, then the value of numerical difference $V_1 - V'_2$ is given by ($I_1R_{01}\cos\varphi + I_1X_{01}\sin\varphi$) for lagging p.f.

$$\boxed{\%\text{regn} = \frac{I_1R_{01}\cos\varphi + I_1X_{01}\sin\varphi}{V_1} \times 100 = v_r\cos\varphi + v_x\sin\varphi} \qquad \text{(approx)}$$

Where, $v_r = \dfrac{I_1R_{01} \times 100}{V_1}$ & $v_x = \dfrac{I_1X_{01} \times 100}{V_1}$

As before, if angle between V_1 and V_2' is not negligible, then

$$\boxed{\%\text{regn} = (v_r\cos\varphi \pm v_x\sin\varphi) + \frac{1}{200}(v_x\cos\varphi \pm v_r\sin\varphi)^2} \qquad \text{(exact)}$$

(3) In the above definitions of regulation, **primary voltage was supposed to be kept constant** and the changes in secondary terminal voltage were considered. As the transformer is loaded, the secondary terminal voltage falls (for a lagging p.f.). Hence, to keep the output voltage constant, the primary voltage must be increased. The rise in primary voltage required to maintain rated output voltage from no-load to full-load at a given power factor expressed as percentage of rated primary voltage gives the regulation of the transformer.
Suppose primary voltage has to be raised from its rated value V_1 to V_1', then

$$\%\,\text{regn.} = \frac{V_1' - V_1}{V_1} \times 100$$

Example 36: A 5 kVA 200/1000 V, 50 Hz, single-phase transformer gave the following test results :
O.C. Test (L.V. Side) : 2000 V, 1.2 A, 90 W
S.C. Test (H.V. Side) : 50 V, 5A, 110 W
(i) Calculate the parameters of the equivalent circuit referred to the L.V. side.
(ii) Calculate the output secondary voltage when delivering 3 kW at 0.8 p.f. lagging, the input primary voltage being 200 V. Find the percentage regulation also.

Solution:
(i) Shunt branch parameters from O.C. test (L.V. side) :

$R_0 = V^2/P_i = 200^2/90 = 444\ \Omega$

$I_w = 200/444 = 0.45$ amp

$I_\mu = (1.22 - 0.452)0.5 = 1.11$ amp

$X_m = 200/1.11 = 180.2\ \Omega$

Or

$$\cos\varphi_0 = \frac{W_0}{V_1 I_0} = \frac{90}{200\times1.2} = 0.375\ \&\ \sin\varphi_0 = 0.927$$

$I_w = I_0 \cos\varphi_0 = 1.2 \times 0.375 = 0.45$ amp, $I_\mu = I_0 \sin\varphi_0 = 1.2 \times 0.927 = 1.11$ amp

$$R_0 = \frac{W_0}{I_w^2} = \frac{90}{0.45^2} = 444\ \Omega,\ (\text{ or } R_0 = \frac{V_1}{I_w} = \frac{200}{0.45} = 444\ \Omega\ )$$

$$X_0 = \frac{V_1}{I_\mu} = \frac{200}{1.11} = 180.18\ \Omega$$

All these are referred to L.V. side.

(ii) Series-branch Parameters from S.C test (H.V side) :
Since the S.C. test has been conducted from H.V. side, the parameters will refer to H.V. side. They should be converted to the parameters referred to L.V. side by transforming them suitably.
From S.C. Test readings, $Z_{02} = 50/5 = 10\ \Omega$
$R = 110/25 = 4.40$ ohms, $X_{02} = (10^2 - 4.4^2)^{0.5} = 8.9\ \Omega$
These are referred to H.V. side.
For referring these to L.V. side, transform these using the ratio of turns, as follows :
$r_1 = 4.40 \times (200/1000)^2 = 0.176\ \Omega$, $x_1 = 8.98 \times (200/1000)^2 = 0.36\ \Omega$
Equivalent circuit can be drawn with R_0 and X_m calculated above and r_1 and x_1 as above.
L.V. Current at rated load= 5000/200 = 25 A
L.V. Current at 3 kW at 0.8 lagging p.f. = (3000/0.80)/200 = 18.75 A
Regulation at this load = $18.75\,(r_1 \cos\varphi + x_1 \sin\varphi) = 18.75\,(0.176 \times 0.80 + 0.36 \times 0.6)$
$= +\,6.69$ Volts $= +\,(6.69/200) \times 100\% = +\,3.345\%$
This is referred to L.V. side, and positive sign means voltage drop.
Regulation in volts ref. to H.V. side = 6.69 × 1000/200 = 33.45 V
With 200 V across primary (i.e. L.V. side), the secondary (i.e. H.V. side)
terminal voltage = 1000 − 33.45 = 966.55 V

Example 37: A-100 kVA transformer has 400 turns on the primary and 80 turns on the secondary. The primary and secondary resistances are 0.3 Ω and 0.01 Ω respectively and the corresponding leakage reactances are 1.1 and 0.035 Ω respectively. The supply voltage is 2200 V. Calculate **(i)** equivalent impedance referred to primary and **(ii)** the voltage regulation and the secondary terminal voltage for full load having a power factor of 0.8 leading.

Solution:

$K = 80/400 = 1/5$

$R_1 = 0.3\ \Omega$, $R_{01} = R_1 + R_2/K^2 = 0.3 + 0.01/(1/5)^2 = 0.55\ \Omega$

$X_{01} = X_1 + X_2/K^2 = 1.1 + 0.035/(1/5)^2 = 1.975\ \Omega$

(i) $Z_{01} = 0.55 + j\,1.975 = 2.05\ \angle 74.44^o$

(ii) The voltage regulation and the secondary terminal voltage for full load can be found by different ways,

(a) $Z_{02} = K^2 Z_{01} = (1/5)^2\,(0.55 + j\,1.975) = (0.022 + j\,0.079)$

N.L. secondary voltage $= KV_1 = (1/5) \times 2200 = 440$ V

$I_2 = S / V_2 = (10 \times 10^3) / 440 = 227.3$ A

F.L. secondary voltage drop $= I_2 (R_{02} \cos\varphi - X_{02} \sin\varphi)$

$= 227.3\,(0.022 \times 0.8 - 0.079 \times 0.6)$

$= -6.77$ V

$$\%\ \text{regn.} = \frac{\text{F.L.secondary voltage drop}}{{}_0V_2} \times 100 = (-6.77/440) \times 100 = -1.54$$

Secondary (terminal) voltage on load $(V_2) = {}_0V_2$ – F.L. secondary voltage drop

$= 440 - (-6.77) =$ **446.77 V**

(b) $I_2 = S / V_2 = (10 \times 10^3) / 440 = 227.3$ A

F.L. secondary voltage drop $= I_2 (R_{02} \cos\varphi - X_{02} \sin\varphi)$

$= 227.3\,(0.022 \times 0.8 - 0.079 \times 0.6)$

$= -6.77$ V

Secondary terminal voltage on load $(V_2) = {}_0V_2$ – F.L. secondary voltage drop

$= 440 - (-6.77) =$ **446.77 V**

$$\%\ \text{regn.} = \frac{{}_0V_2 - V_2}{{}_0V_2} \times 100 = \frac{440 - 446.77}{440} \times 100 = -1.54$$

(c) %regn. as referred to primary,

$I_1 = S / V_1 = (100 \times 10^3) / 2200 = 45.45$ A

F.L. primary drop $= I_1 (R_{01} \cos\varphi - X_{01} \sin\varphi)$
$= 45.45\,(0.55 \times 0.8 - 1.975 \times 0.6)$
$= -33.86$ V

$$\%\text{ regn.} = \frac{\text{F.L.primary drop}}{{}_0V_1} \times 100 = (-33.86 / 2200) \times 100 = -1.54$$

Applied voltage on F.L. $(V_1) = {}_0V_1$ – F.L. primary drop = 2200 – (−33.86) = 2233.86V

Secondary (terminal) voltage on load = KV_1= (1/5)× 2233.86 = **446.77 V**

(d) %regn. as referred to primary,

$I_1 = S / V_1 = (100 \times 10^3) / 2200 = 45.45$ A

F.L. primary drop $= I_1 (R_{01} \cos\varphi - X_{01} \sin\varphi)$
$= 45.45\,(0.55 \times 0.8 - 1.975 \times 0.6)$
$= -33.86$ V

${}_0V_1 = 2200$, $V_1 = {}_0V_1$ – F.L. primary drop = 2200 – (–33.86) = 2233.86 V

$$\%\text{ regn.} = \frac{{}_0V_1 - V_1}{{}_0V_1} \times 100 = \frac{2200 - 2233.86}{2200} \times 100 = -1.54$$

Secondary (terminal) voltage on load = KV_1= (1/5)× 2233.86 = **446.77 V**

Notes:

* %regn value for a transformer is the same whether calculated from primary or secondary side.
* Secondary (terminal) voltage on load (V_2) = K × Applied voltage on F.L. (V_1)

$$V_2 = KV_1$$

* Applied voltage on F.L. (V_1) = Secondary (terminal) voltage on load (V_2) / K

$$V_1 = V_2 / K$$

Example 38: The corrected instrument readings obtained from open and short-circuit tests on 10-kVA, 450/120-V, 50-Hz transformer are :
O.C. test : $V_2 = 120$ V; $I_2 = 4.2$ A; $W_2 = 80$ W; V_2, W_1 and I_2 were read on the l.v.
S.C. test : $V_1 = 9.65$ V; $I_1 = 22.2$ A ; $W_1 = 120$ W − with l.v. winding short-circuited
Compute :
(i) the equivalent circuit (approximate) constants,
(ii) efficiency and voltage regulation for an 80% lagging p.f. load,
(iii) the efficiency at half full-load and 80% lagging p.f. load.

Solution:
It is seen from the O.C. test, that with primary open, the secondary draws a no-load current of 4.2 A. Since K = 120/450 = 4/15,
the corresponding no-load primary current $I_0 = 4.2 \times 4/15 = 1.12$ A.
(i) Now, $V_1 I_0 \cos\varphi_0 = 80 \Longrightarrow \therefore \quad \cos\varphi_0 = 80/450 \times 1.12 = 0.159$

$\therefore \quad \varphi_0 = \cos^{-1}(0.159) = 80.9°$; $\sin\varphi_0 = 0.987$

$I_w = I_0 \cos\varphi_0 = 1.12 \times 0.159 = 0.178$A and $I_\mu = 1.12 \times 0.987 =$ **1.1 A**

$\therefore \quad R_0 = 450/0.178 =$ **2530 Ω** and $X_0 = 450/1.1 =$ **409 Ω**

During S.C. test, instruments have been placed in primary.

$\therefore \; Z_{01} = 9.65/22.2 = 0.435\ \Omega$

$R_{01} = 120/22.22 = 0.243\ \Omega$

$X_{01} = 2\ 2\ 0.435 - 0.243 = 0.361\Omega$

Note: R_0 & X_0 can be calculated from secondary side then dividing it by K^2 to convert them to primary side.

The equivalent circuit is shown in Figure (43).

(ii) Total approximate voltage drop as referred to primary is $I_1(R_{01}\cos\varphi + X_{01}\sin\varphi)$, assuming $\varphi_1 = \varphi_2 = \cos^{-1}(0.8)$.

Now, full-load $I_1 = 10{,}000/450 = 22.2$ A

$\therefore \quad$ Drop $= 22.2\,(0.243 \times 0.8 + 0.361 \times 0.6) = 9.2$ V

Regulation $= 9.2 \times 100/450 = 2.04\%$

F.L. losses $= 80 + 120 = 200$ W;

F.L. output $= 10{,}000 \times 0.8 = 8000$ W

$\eta = 8000/(8000 + 200) = 0.9757$ or **97.57%**

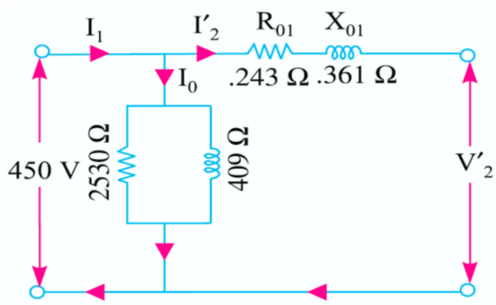


Figure (43)

(iii) Half-load

Note:

$(\text{Total Cu loss})_{F.L.} = I_1^2 R_{01} = I_2^2 R_{02}$

$(\text{Total Cu loss})_{H.F.L.} = (\frac{1}{2} I_1^2) R_{01} = (\frac{1}{2})^2 I_1^2 R_{01} = = \frac{1}{4}\ I_1^2 R_{01} = \frac{1}{4}(\text{Total Cu loss})_{F.L.}$

Or $\quad = (\frac{1}{2} I_2^2) R_{02} = (\frac{1}{2})^2 I_2^2 R_{02} = = \frac{1}{4}\ I_2^2 R_{02} = \frac{1}{4}(\text{Total Cu loss})_{F.L.}$

Iron loss = 80 W; Cu loss = $(1/2)^2 \times 120 = 30$ W; Total losses = 110 W;
Apparent power at Half Full-Load ($S_{H.F.L.}$) = (1/2) × 10000 = 5000 VA
Output = $S_{H.F.L.} \cos\varphi = 5000 \times 0.8 = 4000$ W
∴ $\eta = 4000/(4000 + 110) = 0.9734$ or **97.34%**

Example 39: Consider a 20 kVA, 2200/220 V, 50 Hz transformer. The O.C./S.C. test results are as follows :
O.C. test : 220 V, 4.2 A, 148 W (1.v. side)
S.C. test : 86 V, 10.5 A, 360 W (h.v. side)
Determine the regulation at 0.8 p.f. lagging and at full load. What is the p.f. on S.C. ?

Solution:
It may be noted that O.C. data is not required in this question for finding the regulation.
Since during S.C. test instruments have been placed on the h.v. side i.e. primary side.
∴ $Z_{01} = 86/10.5 = 8.19\ \Omega$; $R_{01} = 360/10.5^2 = 3.26\ \Omega$; $X_{01} = \sqrt{8.19^2 - 3.26^2} = 7.5\ \Omega$
F.L. primary current, $I_1 = 20{,}000/2200 = 9.09$ A
Total voltage drop as referred to primary = $I_1 (R_{01} \cos\varphi + X_{01} \sin\varphi)$
Drop = 9.09 (3.26 × 0.8 + 7.5 × 0.6) = 64.6 V
% age regn. = 64.6 × 100/2200 = **2.9%**
p.f. on short-circuit = R_{01}/Z_{01} = 3.26/8.19 = **0.4 lag**

Or

$$\cos\varphi_{s.c.} = \frac{W_{S.C.}}{V_{S.C.} I_{S.C.}} = \frac{360}{86 \times 10.5} = \mathbf{0.4\ lag}$$

Note: if S.C. test is made on (l.v.) or secondary side, then;
p.f. on short-circuit = R_{02}/Z_{02}

Example 40: A short-circuit test when performed on the h.v. side of a 10 kVA, 2000/400 V single phase transformer, gave the following data ; 60 V, 4 A, 100 W. If the 1.v. side is delivering full load current at 0.8 p.f. lag and at 400 V, find the voltage applied to h.v. side.

Solution:
Here, the test has been performed on the h.v. side i.e. primary side.
$Z_{01} = 60/4 = 15\ \Omega$; $R_{01} = 100/42 = 6.25\ \Omega$; $X_{01} = 2\ 2\ 15 - 6.25 = 13.63\ \Omega$
F.L. $I_1 = 10{,}000/2000 = 5$A
Total transformer voltage drop as referred to primary is,
$I_1(R_{01} \cos\varphi + X_{01} \sin\varphi) = 5\,(6.25 \times 0.8 + 13.63 \times 0.6) = 67$ V
Hence, primary voltage has to be raised from 2000 V to 2067 V in order to compensate for the total voltage drop in the transformer. In that case secondary voltage on load would remain the same as on no-load.

Example 41: A 250/500-V transformer gave the following test results :
Short-circuit test : with low-voltage winding short-circuited : 20 V; 12 A, 100 W
Open-circuit test : 250 V, 1 A, 80 W on low-voltage side.
Determine the circuit constants, insert these on the equivalent circuit diagram and calculate applied voltage and efficiency when the output is 10 A at 500 volt and 0.8 power factor lagging.

Solution:

Open-circuit Test :

$V_1I_0 \cos\varphi_0 = 80$ $\therefore \cos\varphi_0 = 80/250 \times 1 = 0.32$

$I_w = I_0 \cos\varphi_0 = 1 \times 0.32 = 0.32$ A, $I_\mu = \sqrt{1^2 - 0.32^2} = 0.95$ A

$R_0 = V_1/I_w = 250/0.32 = 781.3\ \Omega$, $X_0 = V_1 / I_\mu = 250/0.95 = 263.8\ \Omega$

Short-circuit Test :

As the primary is short-circuited, all values refer to secondary winding.

$\therefore R_{02} = (W_{S.C.} / (I_{2(F.L.)})^2) = (100 / 12^2) = 0.694\ \Omega$

$Z_{02} = 20/12 = 1.667\ \Omega$; $X_{02} = \sqrt{1.667^2 - 0.694^2} = 1.518\ \Omega$

As R_0 and X_0 refer to primary, hence we will transfer these values to primary with the help of transformation ratio.

$K = 500/250 = 2$ $\therefore R_{01} = R_{02}/K^2 = 0.694/4 = 0.174\ \Omega$

$X_{01} = X_{02}/K^2 = 1.518/4 = 0.38\ \Omega$; $Z_{01} = Z_{02}/K^2 = 1.667/4 = 0.417\ \Omega$

The equivalent circuit is shown in Figure (44).

Efficiency

Total Cu loss $= I_2^2R_{02} = 100 \times 0.694 = 69.4$ W ;

Note : if we have Cu loss at any load current (I_a) & we want to find the Cu loss at another current (I_b) then,

$$W_{cua} = I_a^2R \quad \ldots(i)$$
$$W_{cub} = I_b^2R \quad \ldots(ii)$$

Divide equ.(ii) on equ.(i) we get,

$$W_{cub} = \left(\frac{I_b}{I_a}\right)^2 \times W_{cua}$$

Also, $W_{cu(10A)} = \left(\frac{10}{12}\right)^2 \times W_{cu(12A)} = \left(\frac{10}{12}\right)^2 \times 100 = 69.4$ W

Iron loss = 80 W

Total loss = 69.4 + 80 = 149.4 W

$\therefore \eta = \frac{5000 \times 0.8 \times 100}{4000+149.4} =$ **96.42%**

$I_1 = KI_2 = 2 \times 10 = 20$ A

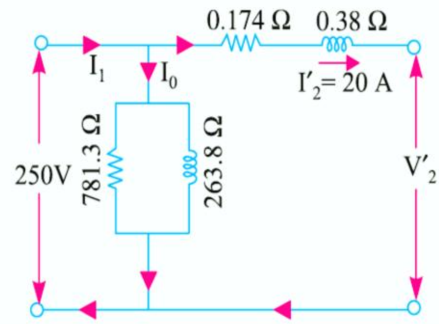


Figure (44)

F.L. primary drop $= I_1 (R_{01} \cos\varphi + X_{01} \sin\varphi) = 20 \times (0.174 \times 0.8 + 0.38 \times 0.6) =$ **7.34 V**

$V_1 = {}_0V_1 +$ F.L. primary drop $= 250 + 7.34 =$ **257.34 V**

Example 42: A 230/230 V, 3 kVA transformer gave the following results :
O.C. Test : 230 V, 2 amp, 100 W
S.C. Test : 15 V, 13 amp, 120 W
Determine the regulation and efficiency at full load 0.80 p.f. lagging.

Solution:
This is the case of a transformer with turns ratio as 1 : 1. Such a transformer is mainly required for isolation.
Rated Current = 3000 / 230 = 13 amp
Cu-losses at rated load = 120 watts, from S.C. test
Core losses = 100 watts, from O.C. test
At full load, VA output = **3000**
At 0.8 lag p.f., Power output = 3000 × 0.8 = 2400 watts
Required efficiency = [2400 / (2400 + 220)] ×100% = 91.6%
From S.C. test, Z =15 / 13 = 1.154 ohms
R = 120 / (15 × 15) = 0.53 ohm, $X = \sqrt{1.154^2 - 0.53^2}$ = 1.0251 ohm
Approximate voltage regulation = IR cos φ + IX sin φ = 13[0.53 × 0.8 + 1.0251 × 0.6]
= 13[0.424 + 0.615] = 13.51 volts
In terms of %, the voltage regulation = (13.51 / 230) × 100% = **5.874%**

Example 43: A 10 kVA, 500/250 V, single-phase transformer has its maximum efficiency of 94% when delivering 90% of its rated output at unity p.f. Estimate its efficiency when delivering its full-load output at p.f. of 0.8 lagging.

Solution:
Rated output at unity p.f. = 10000 W. Hence, 90% of rated output = 9,000 W
Input with 94% efficiency = 9000/0.94 W
Losses = 9000((1/0.94) − 1) = 574 W
At maximum efficiency, variable copper-loss = constant = Core loss = 574/2 = 287 W
At rated current, Let the copper-loss = Pc watts
At 90% load with unity p.f., the copper-loss is expressed as 0.902 × Pc.
Hence, Pc = 287/0.81 = 354 W
Or
$W_{cu2} = 287(10 / 9)^2 = 354W$

(b) Output at full-load, 0.8 lag p.f. = 10,000 × 0.80 = 8000 W
At the corresponding load, Full Load copper-loss = 354 W
Hence, efficiency = 8000/(8000 + 354 + 287) = 0.926 = **92.6%**

Example 44: Resistances and Leakage reactance of 10 kVA, 50 Hz, 2300/230 V single phase distribution transformer are r1 = 3.96 ohms, r2 = 0.0396 ohms, x1 = 15.8 ohms, x2 = 0.158 ohm. Subscript 1 refers to HV and 2 to LV winding (a) transformer delivers rated kVA at 0.8 p.f. Lagging to a load on the L.V. side. Find the H.V. side voltage necessary to maintain 230 V across Load terminals. Also find percentage voltage regulation. (b) Find the power-factor of the rated load current at which the voltage regulation will be zero, hence find the H.V. side voltage.

Solution:

(a) Rated current on L.V. side = 10,000/230 = 43.5 A. Let the total resistance and total leakage reactance be referred to L.V. side. Finally, the required H.V. side voltage can be worked out after transformation.

Total resistance, $r = r_1' + r_2 = 3.96 \times (230/2300)^2 + 0.0396 = 0.0792$ ohms

Total leakage-reactance, $x = x1' + x2 = 15.8 \times (230/2300)^2 + 0.158 = 0.316$ ohm

For purpose of calculation of voltage-magnitudes, approximate formula for voltage regulation can be used. For the present case of 0.8 lagging p.f.

$V_1' = V_2 + I\,[r\cos\varphi + x\sin\varphi] = 230 + 43.5\,[(0.0792 \times 0.8) + (0.316 \times 0.6)]$

$= 230 + 43.5\,[0.0634 + 0.1896] = 230 + 11 = 241$ volts

Hence, $V_1 = 241 \times (2300/230) = 2410$ volts.

It means that H.V. side terminal voltage must be 2410 for keeping 230 V at the specified load.

(b) Approximate formula for voltage regulation is : $V_1' - V_2 = I\,[r\cos\varphi \pm x\sin\varphi]$

With Lagging p.f., +ve sign is retained. With leading power-factor, the −ve sign is applicable. For the voltage-regulation to be zero, only leading P.f. condition can prevail.

Thus, $r\cos\varphi - x\sin\varphi = 0$

Or $\tan\varphi = r/x = 0.0792/0.316 = 0.25$

or $\varphi = 14^o$, $\cos\varphi = 0.97$ leading

Corresponding $\sin\varphi = \sin 14^o =$ **0.243**

H.V. terminal voltage required is 2300 V to maintain 230 V at Load, since Zero regulation condition is under discussion.

Example 45: A 5 kVA, 2200/220 V, single-phase transformer has the following parameters.
H.V. side : $r_1 = 3.4$ ohms, $x_1 = 7.2$ ohms
L.V. side : $r_2 = 0.028$ ohms, $x_2 = 0.060$ ohms
Transformer is made to deliver rated current at 0.8 lagging P.f. to a load connected on the L.V. side. If the load voltage is 220 V, calculate the terminal voltage on H.V. side

Solution:

$K = 1/10$

$R_{01} = 3.4 + (1/10)^2 \times 0.028 = 6.2\ \Omega$

$X_{01} = 7.2 + (1/10)^2 \times 0.06 = 13.2\ \Omega$

$I_2 = 5000/220 = 22.7$ A

$I_1 = KI_2 = 22.7 / 10 = 2.27$ A

Primary voltage drop on F.L. $= I_1(R_{01}\cos\varphi + X_{01}\sin\varphi)$
$= 2.27 \times (6.2 \times 0.8 + 13.2 \times 0.6) = 29.24$ V

Terminal voltage on F.L. $= {}_0V_1$ + Primary voltage drop on F.L.
$2200 + 29.24 = 2229.24$ V

Example 46: A 4-kVA, 200/400 V, single-phase transformer takes 0.7 amp and 65 W on Open circuit. When the low-voltage winding is short-circuited and 15 V is applied to the high-voltage terminals, the current and power are 10 A and 75 W respectively. Calculate the full-load efficiency at unity power factor and full-load regulation at 0.80 power-factor lagging.

Solution:

At a load of 4 kVA, the rated currents are :
L.V. side : 4000/200 = 20 amp
And H.V. side : 4000/400 = 10 amp
From the test data, full-load copper-loss = 75 W
And constant core-loss = 65 W
From S.C. test, Z = 15/10 = 1.5 ohms
R = 75/100 = 0.75 ohm
Hence x = 1.52 − 0.752 = 1.30 ohms
All these series-parameters are referred to the H.V. side, since the S.C. test has been conducted from H.V. side.
Full-load efficiency at unity p.f. = 4000 / (4000 + 65 + 75) = 0.966 = 96.6%
Full load voltage regulation at 0.80 lagging p.f. $= I_r \cos\varphi + I_x \sin\varphi$
$= 10\ (0.75 \times 0.80 + 1.30 \times 0.60)$
$= 16.14$ Volts

Thus, due to loading, H.V. side voltage will drop by 16.14 volts (i.e. terminal voltage for the load will be 383.86 volts), when L.V. side is energized by 200-V source.

19. Percentage Resistance, Reactance and Impedance

These quantities are usually measured by the voltage drop at full-load current expressed as a percentage of the normal voltage of the winding on which calculations are made.

(i) Percentage resistance at full-load

$$\% \, R = \frac{I_1 R_{01}}{V_1} \times 100 = \frac{I_1^2 R_{01}}{V_1 I_1} \times 100$$

$$= \frac{I_2^2 R_{02}}{V_2 I_2} \times 100 = \% \text{ Cu loss at F.L.}$$

$$\%R = \%\text{Cu loss} = v_r$$

(ii) Percentage reactance at full-load

$$\% \, X = \frac{I_1 X_{01}}{V_1} \times 100 = \frac{I_2 X_{02}}{V_2} \times 100 = v_x$$

(iii) Percentage impedance at full-load

$$\% \, Z = \frac{I_1 Z_{01}}{V_1} \times 100 = \frac{I_2 Z_{02}}{V_2} \times 100$$

(iv) $\% \, Z = \sqrt{\%R^2 + \%X^2}$

It should be noted from above that the reactances and resistances in ohm can be obtained thus :

$$R_{01} = \frac{\% \, R \times V_1}{100 \times I_1} = \frac{\% \text{ Cu loss } \times V_1}{100 \times I_1}, \text{ Similarly } \quad R_{02} = \frac{\% \, R \times V_2}{100 \times I_2} = \frac{\% \text{ Cu loss } \times V_2}{100 \times I_2}$$

$$X_{01} = \frac{\% \, X \times V_1}{100 \times I_1} = \frac{v_x \times V_1}{100 \times I_1}, \quad \text{Similarly } \quad X_{02} = \frac{\% \, X \times V_2}{100 \times I_2} = \frac{v_x \times V_2}{100 \times I_2}$$

$$Z_{01} = \frac{\% \, Z \times V_1}{100 \times I_1}, \quad \text{Similarly } \quad Z_{02} = \frac{\% \, Z \times V_2}{100 \times I_2} =$$

It may be noted that percentage resistance, reactance and impedance have the same value whether referred to primary or secondary.

Example 47: A 3300/230 V, 50-kVA, transformer is found to have impedance of 4% and a Cu loss of 1.8% at full-load. Find its percentage reactance and also the ohmic values of resistance, reactance and impedance as referred to primary. What would be the value of primary short-circuit current if primary voltage is assumed constant ?

Solution:

$$\%X = \sqrt{\%Z^2 - \%R^2} = \sqrt{4^2 - 1.8^2} = 3.57\% \quad , \quad (\text{Cu loss} = \%R)$$

Full load I_1 = 50,000/3300 = 15.2 A (assuming 100% efficiency). Considering primary winding, we have

$$R_{01} = \frac{\%R \times V_1}{100 \times I_1} = \frac{1.8 \times 3300}{100 \times 15.2} = \mathbf{3.91\ \Omega}$$

$$X_{01} = \frac{\%X \times V_1}{100 \times I_1} = \frac{3.57 \times 3300}{100 \times 15.2} = \mathbf{7.76\ \Omega}$$

$$Z_{01} = \frac{\%Z \times V_1}{100 \times I_1} = \frac{4 \times 3300}{100 \times 15.2} = \mathbf{8.7\ \Omega}$$

Now,

$$I_{s.c.} = \frac{V_1}{Z_{01}} \quad \& \quad Z_{01} = \frac{\%Z \times V_1}{100 \times I_1}$$

$$I_{s.c.} = \frac{V_1 \times 100 \times I_1}{\%Z \times V_1} = \frac{100 \times I_1}{\%Z}$$

$$\therefore \quad \frac{I_{s.c.}}{I_1} = \frac{100}{\%Z}$$

$$\therefore \quad \frac{I_{s.c.}}{15.2} = \frac{100}{4}$$

$$\therefore \quad I_{s.c.} = \mathbf{380\ A}$$

Example 48: The full-load copper loss on the h.v. side of a 100-kVA, 11000/317-V, 1-phase transformer is 0.62 kW and on the L.V. side is 0.48 kW. **(i)** Calculate R_1, R_2 and R'_2 in ohms **(ii)** the total reactance is 4 per cent, find X_1, X_2 and X'_2 in ohms if the reactance is divided in the same proportion as resistance.

Solution:

(i) F.L. $I_1 = 100 \times 10^3/11000 = 9.1$ A. F.L. $I_2 = 100 \times 10^3/317 = 315.5$ A

Now,

$I_1^2 R_1 = 0.62$ kW $\Rightarrow$ $R_1 = 620/9.1^2 = 7.5\ \Omega$

$I_2^2 R_2 = 0.48$ Kw $\Rightarrow$ $R_2 = 480/315.5^2 = 0.00482\ \Omega$

$R_2' = R_2/K^2 = 0.00482 \times (11{,}000/317)^2 = 5.8\ \Omega$

$$\%\text{ reactance} = \frac{I_1 \times X_{01}}{V_1} \times 100\%$$

$$\therefore \quad 4 = \frac{9.1 \times X_{01}}{11000} \times 100 \Longrightarrow X_{01} = 48.4\ \Omega$$

Given that, $R_1/R_2' = X_1/X_2' \Rightarrow (7.5/5.8) = X_1/X_2'$

$\therefore X_1 = 1.29\, X_2'$

$X_{01} = X_1 + X_2' \Rightarrow \therefore 48.4 = X_1 + X_2'$

$\therefore 48.4 = 1.29\, X_2' + X_2' \Rightarrow X_2' = 21.14\ \Omega$

$X_1 = 1.29\, X_2' = 1.29 \times 21.14 = 27.26\ \Omega$

Or from secondary side,

$$\%\ \text{reactance} = \frac{I_2 \times X_2}{V_2} \times 100\%$$

$$\therefore \quad 4 = \frac{315.5 \times X_{02}}{317} \times 100 \Rightarrow X_{02} = 0.04\ \Omega$$

$R'_1 = K^2 R_1 = (317 / 11000)^2 \times 7.5 = 0.00623\ \Omega$

Given that, $R_2/R_1' = X_2/X_1' \Rightarrow (0.00482 / 0.00623) = X_2/X_1'$

$X_2 = 0.774\, X_1'$

$X_{02} = X_2 + X_1' \Rightarrow \therefore 0.04 = X_2 + X_1'$

$\therefore 0.04 = 0.774\, X_1' + X_1' \Rightarrow X_1' = 0.0226\ \Omega \quad \& \quad X_2 = 0.01745\ \Omega$

$$X_1 = X_1' / K^2 = \frac{0.0226}{\left(\frac{317}{11000}\right)^2} = 27.22\ \Omega$$

$$X_2' = X_2 / K^2 = \frac{0.01745}{\left(\frac{317}{11000}\right)^2} = 21.01\ \Omega$$

Example 49: transformer has a reactance drop of 5% and a resistance drop of 2.5%. Find the lagging power factor at which the voltage regulation is maximum and the value of this regulation.

Solution:

The percentage voltage regulation (μ) is given by, $\mu = v_r \cos\varphi + v_x \sin\varphi$

where v_r is the percentage resistive drop and v_x is the percentage reactive drop.

Differentiating the above equation, we get, $\dfrac{d\mu}{d\varphi} = -v_r \sin\varphi + v_x \cos\varphi$

For regulation to be maximum, $d\mu/d\varphi = 0$

$\therefore \quad -v_r \sin\varphi + v_x \cos\varphi = 0$

or $\quad \tan\varphi = v_x/v_r = 5/2.5 = 2$

$\therefore \quad \varphi = \tan{-1}\ (2) = 63.5°$

Now, $\cos\varphi = 0.45$ and $\sin\varphi = 0.892$

Maximum percentage regulation = $(2.5 \times 0.45) + (5 \times 0.892) = 5.585$

Maximum percentage regulation is **5.585** and occurs at a power factor of **0.45** (lag).

Example 50: Calculate the percentage voltage drop for a transformer with a percentage resistance of 2.5% and a percentage reactance of 5% of rating 500 kVA when it is delivering 400 kVA at 0.8 p.f. lagging.

Solution:

$$\%\ \text{drop} = \frac{(\%R)I\cos\emptyset}{I_f} + \frac{(\%X)I\sin\emptyset}{I_f}$$

where I_f is the full-load current and I the actual current.

$$\therefore\ \%\ \text{drop} = \frac{(\%R)\text{kW}}{\text{kVA rating}} + \frac{(\%X)\text{kVAR}}{\text{kVA rating}}$$

In the present case, kW = 400 × 0.8 = 320 and kVAR = 400 × 0.6 = 240

$$\therefore\ \%\ \text{drop} = \frac{2.5 \times 320}{500} + \frac{5 \times 240}{500} = \mathbf{4\%}$$

Example 51: A 20-kVA, 2200/220-V, 50-Hz distribution transformer is tested for efficiency and regulation as follows :
O.C. test : 220 V 4.2 A, 148 W –l.v side
S.C. test : 86 V 10.5 A, 360 W – l.v. side
Determine **(a)** core loss **(b)** equivalent resistance referred to primary **(c)** equivalent resistance referred to secondary **(d)** equivalent reactance referred to primary **(e)** equivalent reactance referred to secondary **(f)** regulation of transformer at 0.8 p.f. lagging current **(g)** efficiency at full-load and half the full-load at 0.8 p.f. lagging current.

Solution:

(a) No-load primary input is practically equal to the core loss.
Hence, core loss as found from no-load test, is **148 W.**

(b) From S.C. test, $R_{01} = 360/10.5^2 =$ **3.26 Ω**

(c) $R_{02} = K^2R_{01} = (220/2200)^2 \times 3.26 =$ **0.0326 Ω**

(d) $Z_{10} = V_{s.c.} / I_{s.c.} = 86 / 10.5 = 8.19\ \Omega$
$X_{01} = \sqrt{(8.19^2 - 3.26^2)} =$ **7.51 Ω**

(e) $X_{02} = K^2 X_{01} = (220/2200)^2 \times 7.51 =$ **0.0751 Ω**

(f) We will find the rise in primary voltage necessary to maintain the output terminal voltage constant from no-load to full-load.
Rated primary current = 20,000/2200 = 9.1 A
F.L. primary drop = $I_1 (R_{01} \cos\varphi + X_{01} \sin\varphi) = 9.1 (3.26 \times 0.8 + 7.51 \times 0.6) = 64.74$ V

$$\% \text{ regn.} = \frac{\text{F.L.primary drop}}{0V_1} \times 100 = (64.74 / 2200) \times 100 = \mathbf{2.94\%}$$

Or from secondary side,

$I_2 = I_1 / K = 9.1 / (220 / 2200) = 91$ A

F.L. secondary drop $= I_2 (R_{02} \cos\varphi + X_{02} \sin\varphi)$

$= 91 (0.0326 \times 0.8 + 0.0751 \times 0.6)$

$= 6.474$ V

$$\% \text{ regn.} = \frac{\text{F.L.secondary drop}}{0V_2} \times 100 = (6.474 / 220) \times 100 = \mathbf{2.94\%}$$

(g) Core loss = 1.48 W. It will be the same for all loads.

Cu loss at full load $= I_1^2 R_{01} = 9.1^2 \times 3.26 = 270$ W

Cu loss at half full-load $= 4.55^2 \times 3.26 = 67.5$ W (or F.L. Cu loss/4)

Output power $= 20000 \times 0.8 = 16000$ KW

Input power = output power + total losses = 16000 + 148 + 270 = 16418 KW

$$\therefore\ \eta \text{ at full-load} = \frac{16000}{16418} \times 100\% = \mathbf{97.45\%}$$

At half full-load,

$$\text{Output power} = (\frac{1}{2} \times 20000) \times 0.8 = 8000 \text{ KW}$$

Input power = output power + total losses $= 8000 + 148 + ((\frac{1}{2})^2 \times 270) = 8215.5$ KW

$$\therefore\ \eta \text{ at half-load} = \frac{8000}{8215.5} \times 100\% = \mathbf{97.38\%}$$

Example 52: Calculate the regulation of a transformer in which the ohmic loss is 1% of the output and the reactance drop is 5% of the voltage, when the power factor is (i) 0.80 Lag (ii) unity (iii) 0.80 Leading.

Solution:

When 1% of output is the ohmic loss, p.u. resistance of the transformer, $\varepsilon_r = 0.01$

When 5% is the reactance drop, p.u. reactance of the transformer $\varepsilon_x = 0.05$

(i) Per Unit regulation of the transformer at full-load, 0.8 Lagging p.f.

$= 0.01 \times \cos\varphi + 0.05 \times \sin\varphi = 0.01 \times 0.8 + 0.05 \times 0.06 = 0.038$ or 3.8%

(ii) Per Unit regulation at unity p.f. $= 0.01 \times 1 = 0.01$ or 1%

(iii) Per Unit regulation at 0.08 Leading p.f. $= 0.01 \times 0.8 - 0.05 \times 0.6 = -0.022$ or -2.2%

Example 53: The maximum efficiency of a 500 kVA, 3300/500 V, 50 Hz, single phase transformer is 97% and occurs at 3/4th full-load u.p.f. If the impedance is 10% calculate the regulation at full load, 0.8 p.f. Lag.

Solution:

At u.p.f. with 3/4th full load, the output of the transformer = 500 × 0.75 × 1 kW = 375 kW

$0.97 = 375 / (375 + 2P_i)$

where Pi = core loss in kW, at rated voltage.

At maximum efficiency, $x^2 P_c = P_i$

$(0.75)^2 P_c = P_i$

where x = 0.75, i.e. 3/4th which is the fractional loading of the transformer

P_c = copper losses in kW, at rated current

$P_i = \frac{1}{2}\left\{375 \times \left(\frac{1}{0.97} - 1\right)\right\} = 5.8$ KW

$P_c = 5.8/(0.75)^2 = 10.3$ kW

Full load current in primary (H.V.) winding = (500 × 1000) / 3300 = **151.5 amp**

Total winding resistance ref. to primary = (10.3 × 1000) / $(151.5)^2$ = 0.44876 ohm

ε_r = % resistance = $\frac{(151.5 \times 0.44876)}{3300} \times 100\%$ = **2.06%**

ε_z = % Impedance = 10%

ε_x = % reactance = $\sqrt{100 - 4.244}$ = **9.7855%**

By Approximate formula at 0.8 p.f. lag,

% regulation = $\varepsilon_r \cos\varphi + \varepsilon_x \sin\varphi$ = 2.06 × 0.8 + 9.7855 × 0.6 = 1.648 + 5.87 = **7.52 %**

Example 54: A transformer has copper-loss of 1.5% and reactance-drop of 3.5% when tested at full-load. Calculate its full-load regulation at (i) u.p.f. (ii) 0.8 p.f. Lagging and (iii) 0.8 p.f. Leading.

Solution:

The test-data at full-load gives following parameters :

p.u. resistance = 0.015, p.u. reactance = 0.035

(i) Approximate Voltage − Regulation at unity p.f. full load

= 0.015 cos φ + 0.035 sin φ = 0.015 per unit = **1.5%**

(ii) Approximate Voltage − Regulation at 0.80 Lagging p.f.

= (0.015 × 0.8) + (0.035 × 0.6) = 0.033 per unit = **3.3%**

(iii) Approximate Voltage Regulation at 0.8 leading p.f.

$= I_r \cos\varphi - I_x \sin\varphi$

= (0.015 × 0.8) − (0.035 × 0.6) = − 0.009 per unit = **− 0.9%**

20. Losses in a Transformer

In a static transformer, there are no friction or windage losses. Hence, the only losses occurring are:

(i) Core or Iron Loss: It includes both hysteresis loss and eddy current loss. Because the core flux in a transformer remains practically constant for all loads (its variation being 1 to 3% from no-load to full-load). The core loss is practically the same at all loads. Hysteresis loss $W_h = \eta B_{max}^{1.6} f V$ watt; eddy current loss $W_e = KB_{max}^2 f^2 t^2 V^2$ watt
These losses are minimized by using steel of high silicon content for the core and by using very thin laminations. Iron or core loss is found from the O.C. test. **The input of the transformer when on no load measures the core loss**.

(ii) Copper loss. This loss is due to the ohmic resistance of the transformer windings. Total Cu loss = $I_1^2R_1 + I_2^2R_2 = I_1^2R_{01} = I_2^2R_{02}$. It is clear that Cu loss is proportional to $(\text{current})^2$ or kVA^2. In other words, Cu loss at half the full-load is one-fourth of that at full-load. The value of Cu loss is found from the short-circuit test.

21. Efficiency of a Transformer

As is the case with other types of electrical machines, the efficiency of a transformer at a particular load and power factor is defined as the output divided by the input–the two being measured in the same units (either watts or kilowatts).

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}$$

But a transformer being a highly efficient piece of equipment, has very small loss, hence it is impractical to try to measure transformer, efficiency by measuring input and output. These quantities are nearly of the same size. A better method is to determine the losses and then to calculate the efficiency from;

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{losses}} = \frac{\text{Output}}{\text{Output} + \text{cu loss} + \text{iron loss}}$$

Or $$\eta = \frac{\text{Input} - \text{Losses}}{\text{Input}} = 1 - \frac{\text{Losses}}{\text{Input}}$$

It may be noted here that efficiency is based on power output in watts and not in volt-amperes, although losses are proportional to VA. Hence, at any volt-ampere load, the efficiency depends on power factor, being maximum at a power factor of unity.
Efficiency can be computed by determining core loss from no-load or open-circuit test and Cu loss from the short-circuit test.

22. Condition for Maximum Efficiency

Cu loss = $I_1^2 R_{01}$ or Cu loss = $I_2^2 R_{02}$

Iron loss = Hysteresis loss + Eddy current loss = $W_h + W_e = W_i$

Considering primary side,

Primary input = $V_1 I_1 \cos\varphi_1$

$$\eta = \frac{V_1 I_1 \cos\varphi_1 - \text{losses}}{V_1 I_1 \cos\varphi_1} = \frac{V_1 I_1 \cos\varphi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos\varphi_1}$$

$$= 1 - \frac{I_1 R_{01}}{V_1 \cos\varphi_1} - \frac{W_i}{V_1 I_1 \cos\varphi_1}$$

Differentiating both sides with respect to I_1, we get

$$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1 \cos\varphi_1} + \frac{W_i}{V_1 I_1^2 \cos\varphi_1}$$

For η to be maximum, $\frac{d\eta}{dI_1} = 0$ Hence, the above equation becomes

$$\frac{R_{01}}{V_1 \cos\varphi_1} = \frac{W_i}{V_1 I_1^2 \cos\varphi_1},$$ Or, $W_i = I_1^2 R_{01}$ Or, $W_i = I_2^2 R_{02}$

Or $\boxed{\text{Cu loss} = \text{Iron loss}}$

The output current corresponding to maximum efficiency is $\boxed{I_2 = \sqrt{\frac{W_i}{R_{02}}}}$.

It is this value of the output current which will make the Cu loss equal to the iron loss. By proper design, it is possible to make the maximum efficiency occur at any desired load.

Note: **(i)** If we are given iron loss and full load Cu loss, then the load at which two losses would be equal (i.e. corresponding to maximum efficiency) is given by

$$= \text{Full load} \times \sqrt{\left(\frac{\text{Iron loss}}{\text{F.L.cu loss}}\right)}$$

(ii) The efficiency at any load is given by

$$\eta = \frac{x \times \text{full-load kVA} \times \text{p.f.}}{(x \times \text{full-load kVA} \times \text{p.f.}) + x^2 \times w_{cu} + w_i} \times 100$$

Where x = ratio of actual to full-load KVA; W_i = iron loss in kW; W_{cu} = Cu loss in kW.

Example 55: In a 25-kVA, 2000/200 V, single-phase transformer, the iron and full-load copper losses are 350 and 400 W respectively. Calculate the efficiency at unity power factor on **(i)** full load **(ii)** half full-load.

Solution:

(i) Full-load Unity p.f.

Total loss = 350 + 400 = 750 W

F.L. output at u.p.f. = 25 × 1 = 25 kW ; η = 25/25.75 = 0.97 or **97%**

(ii) Half F.L. Unity p.f.

Cu loss = $400 \times (1/2)^2$ = 100 W.

Iron loss remains constant at 350 W, Total loss = 100 + 350 = 450 W.

Half-load output at u.p.f. = 12.5 kW

$\therefore$ η= 12.5/(12.5 + 0.45) = **96.52%**

Example 56: If P_1 and P_2 be the iron and copper losses of a transformer on full-load, find the ratio of P_1 and P_2 such that maximum efficiency occurs at 75% full-load.

Solution:

At F.L.($I_1 = I_{F.L.}$),

$$(P_2)_{at\ F.L.} = I_1^2 R \qquad \ldots(i)$$

At 75% F.L.($I_2 = 0.75 I_1$),

$$(P_2)_{at\ 75\%\ F.L.} = I_2^2 R$$
$$= (0.75 I_1)^2 R \qquad \ldots(ii)$$

Divide equ.(i) on (equ.(ii) we get,

$$(P_2)_{at\ 75\%\ F.L} = (P_2)_{at\ F.L.} \times \left(\frac{0.75 I_1}{I_1}\right)^2 = 0.75^2 (P_2)_{at\ F.L.} = \left(\frac{9}{16}\right)(P_2)_{at\ F.L}$$

At maximum efficiency, it equals the iron loss P_1 which remains constant throughout. Hence, at maximum efficiency.

$$P_1 = (P_2)_{at\ 75\%\ F.L} = \left(\frac{9}{16}\right)(P_2)_{at\ F.L}$$

$$\therefore \frac{P_1}{(P_2)_{at\ F.L.}} = \frac{9}{16}$$

Example 57: A 11000/230 V, 150-kVA, 1-phase, 50-Hz transformer has core loss of 1.4 Kw and F.L. Cu loss of 1.6 kW. Determine
(i) the kVA load for max. efficiency and value of max. efficiency at unity p.f.
(ii) the efficiency at half F.L. 0.8 p.f. leading

Solution:

(i) Load kVA corresponding to maximum efficiency is

$$= \text{Full load} \times \sqrt{\left(\frac{\text{Iron loss}}{\text{F.L.cu loss}}\right)} = 150 \times \sqrt{\left(\frac{1.6}{1.4}\right)} = 160.36\ \text{KVA} \cong 160\ \text{KVA}$$

Since Cu loss equals iron loss at maximum efficiency, total loss = 1.4 + 1.4 = 2.8 kW ;

output = S cosφ = 160 × 1 = 160 kW

input = output + total losses = 160 + 2.8 = 162.8 kW

η_{max} = 160/162.8 = 0.982 or **98.2%**

(ii) Cu loss at half full-load = 1.6 × $(1/2)^2$ = 0.4 kW ;

Total loss = 1.4 + 0.4 = 1.8 kW

Half F.L. output at 0.8 p.f.= (150/2) × 0.8 = 60 kW

∴ Efficiency = 60/(60 + 1.8) = 0.97 or **97%**

Example 58: A 5-kVA, 2,300/230-V, 50-Hz transformer was tested for the iron losses with normal excitation and Cu losses at full-load and these were found to be 40 W and 112 W respectively. Calculate the efficiencies of the transformer at 0.8 power factor for the following kVA outputs :

1.25 2.5 3.75 5.0 6.25 7.5

Plot efficiency vs kVA output curve.

Solution:

F.L. Cu loss = 112 W ; Iron loss = 40 W

(i)

Note: $W_{cu2} = W_{cu1} \times \left(\frac{I_2}{I_1}\right)^2 = W_{cu1} \times \left(\frac{S_2}{S_1}\right)^2$

Cu loss at 1.25 kVA = $W_{cu1} \times \left(\frac{S_2}{S_1}\right)^2$ = 112 × $(1.25/5)^2$ = 7 W

Total loss = 40 + 7 = 47 W

Output = 1.25 × 0.8 = 1 kW = 1,000 W

η = 100 × 1,000/1,047 = **95.51 %**

(ii) Cu loss at 2.5 kVA = $W_{cu1} \times \left(\frac{S_2}{S_1}\right)^2$ = 112 × (2.5/5)^2 = 28 W

Total loss = 40 + 28 = 68 W
Output = 2.5 × 0.8 = 2 kW
η = 2,000 × 100/2,068 = **96.71%**

(iii) Cu loss at 3.75 KVA = $W_{cu1} \times \left(\frac{S_2}{S_1}\right)^2$ = 112 × (3.75/5)^2 = 63 W

Total loss = 40 + 63 = 103 W
η = 3,000 × 100/3,103 = **96.68 %**

(iv) Cu loss at 5 KVA = 112 W
Total loss = 152 W = 0.152 kW
Output = 5 × 0.8 = 4 kW
η = 4 × 100/4.142 = **96.34 %**

(v) Cu loss at 6.25 KVA = $W_{cu1} \times \left(\frac{S_2}{S_1}\right)^2$ = 112 × (6.25/5)^2 = 175 W

Total loss = 125 W = 0.125 kW ; Output = 6.25 × 0.8 = 5 kW
η = 5 × 100/5.215 = **95.88 %**

(vi) Cu loss at 7.5 kVA = $W_{cu1} \times \left(\frac{S_2}{S_1}\right)^2$ = 112 × (7.5/5)^2 = 252 W

Total loss = 292 W = 0.292 kW ; Output = 7.5 × 0.8 = 6 kW
η = 6 × 100/6.292 = **95.36 %**

The curve is shown in Figure (45).

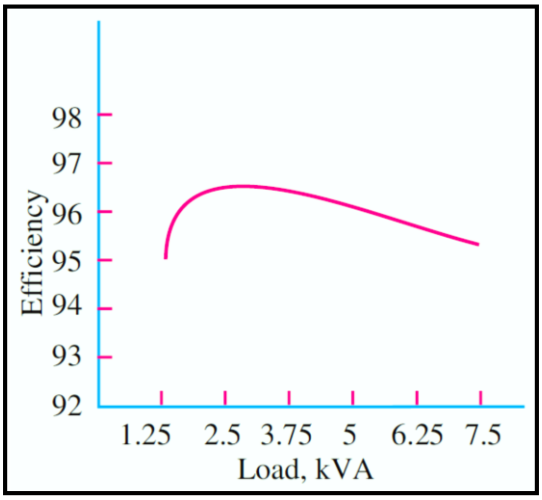


Figure (45)

Example 59: A 200-kVA transformer has an efficiency of 98% at full load. If the max. efficiency occurs at three quarters of full-load, calculate the efficiency at half load. Assume negligible magnetizing current and p.f. 0.8 at all loads.

Solution:

As given, the transformer has a F.L. efficiency of 98 % at 0.8 p.f.

F.L. output = 200 × 0.8 = 160 kW

F.L. input = 160/0.98 = 163.265 kW

F.L. losses = 163.265 − 160 = 3.265 kW

This loss consists of F.L. Cu loss x and iron loss y.

∴ x + y = 3.265 kW ...(i)

It is also given that η_{max} occurs at three quarters of full-load when Cu loss becomes equal to iron loss.

∴ Cu loss at 75 % (or three quarters) of F.L. = x $(3/4)^2$ =9 x/16

Since y remains constant, hence 9 x/16 = y ...(ii)

Substituting the value of y in Eqn. (i), we get,

x + 9 x/16 = 3265 or x = 2090 W; y = 1175 W

Half-load Unity p.f.

Cu loss = 2090 × $(1/2)^2$ = 522 W ;

total loss = 522 + 1175 = 1697 W

Output = 100 × 0.8 = 80 kW ;

η = 80/81.697 = 0.979 or **97.9 %**

Example 60: A 25-kVA, 1-phase transformer, 2,200 volts to 220 volts, has a primary resistance of 1.0 Ω and a secondary resistance of 0.01 Ω. Find the equivalent secondary resistance and the full-load efficiency at 0.8 p.f. if the iron loss of the transformer is 80% of the full-load Cu loss.

Solution:

K = 220/2,200 = 1/10 ;

$R_{02} = R_2 + K^2 R_1$ = 0.01 + 1/100 = **0.02 Ω**

Full-load I_2 = 25,000/220 = 113.6 A ;

F.L. Cu loss = $I_2^2R_{02}$ = 113.6^2 × 0.02 = 258 W.

Iron loss = 80% of 258 = 206.4 W ;

Total loss = 258 + 206.4 = 464.4 W

F.L. output = 25 × 0.8 = 20 kW = 20,000 W

Full-load η = 20,000 × 100/(20,000 + 464.4) = **97.7 %**

Example 61: A 4-kVA, 200/400-V, 1-phase transformer has equivalent resistance and reactance referred to low-voltage side equal to 0.5 Ω and 1.5 Ω respectively. Find the terminal voltage on the high-voltage side when it supplies 3/4th full-load at lagging power factor of 0.8, the supply voltage being 220 V. Hence, find the output of the transformer and its efficiency if the core losses are 100 W.

Solution:

Obviously, primary is the low-voltage side and the secondary, the high voltage side.

Here, $R_{01} = 0.5\ \Omega$ and $X_{01} = 1.5\ \Omega$. These can be transferred to the secondary side with the help of the transformation ratio.

$K = 400/200 = 2$; $R_{02} = K^2 R_{01} = 2^2 \times 0.5 = 2\ \Omega$; $X_{02} = K^2 X_{01} = 4 \times 1.5 = 6\ \Omega$

Secondary current when load is 3/4 the, full-load is $= (1{,}000 \times 4 \times 3/4)/400 = 7.5$ A

Total drop as referred to transformer secondary is $= I_2 (R_{02} \cos\varphi + X_{02} \sin\varphi)$

$= 7.5\,(2 \times 0.8 + 6 \times 0.6) = 39$ V

∴ Terminal voltage on high-voltage side under given load condition is = 400 − 39 = **361 V**

Cu loss $= I_2^2 R_{02} = 7.5^2 \times 2 = 112.5$ W;

Iron loss = 100 W

Total loss = 212.5 W

output = (4 × 3/4) × 0.8 = **2.4 kW**

Input = 2,400 + 212.5 = 2,612.5 W

$\eta = 2{,}400 \times 100/2{,}612.5 =$ **91.87 %**

Example 62: A 20-kVA, 440/220 V, I-φ, 50 Hz transformer has iron loss of 324 W. The Cu loss is found to be 100 W when delivering half full-load current. Determine (i) efficiency when delivering full-load current at 0.8 lagging p.f. and (ii) the percent of full-load when the efficiency will be maximum.

Solution:

F.L. Cu loss $= 2^2 \times 100 = 400$ W ; Iron loss = 324 W

(i) F.L. efficiency at 0.8 p.f. $= \dfrac{20 \times 0.8}{(20 \times 0.8) + 0.724} \times 100$ = **95.67 %**

(ii) $\dfrac{\text{kVA for maximum}}{\text{F.L.kVA}} = \sqrt{\left(\dfrac{\text{Iron loss}}{\text{F.L.cu loss}}\right)} = \sqrt{\left(\dfrac{324}{400}\right)} = 0.9$

Hence, efficiency would be maximum at **90** % of F.L.

Example 63: Consider a 4-kVA, 200/400 V single-phase transformer supplying full-load current at 0.8 lagging power factor. The O.C./S.C. test results are as follows :
O.C. test : 200 V, 0.8 A, 70 W (I.V. side)
S.C. test : 20 V, 10 A, 60 W (H.V. side)
Calculate efficiency, secondary voltage and current into primary at the above load. Calculate the load at unity power factor corresponding to maximum efficiency.

Solution:

Full-load, I_2 = 4000/400 = 10 A

It means that S.C. test has been carried out with full secondary flowing. Hence, 60 W represents full-load Cu loss of the transformer.

Total F.L. losses = 60 + 70 = 130 W ; F.L. output = 4 × 0.8 = 3.2 kW

F.L. η = 3.2/3.33 = 0.96 or **96 %**

S.C. Test

$Z_{02} = 20/10 = 2\ \Omega$; $I_2^2 R_{02} = 60$ or $R_{02} = 60/10^2 = 0.6\ \Omega$; $X_{02} = \sqrt{2^2 - 0.6^2} = 1.9\ \Omega$

Transformer voltage drop as referred to secondary = $I_2 (R_{02} \cos\varphi + X_{02} \sin\varphi)$
= 10 (0.6 × 0.8 + 1.9 × 0.6) = 16.2 V

∴ V_2 = 400 − 16.2 = 383.8 V; Primary current = 4000/200 = 20 A

kVA corresponding to $\eta_{max} = 4 \times 70 / 60 = 4.32$ kVA

∴ Load at u.p.f. corresponding to η_{max} = 4.32 × 1 = **4.32 kW**

Example 64: A 600 kVA, 1-phase transformer has an efficiency of 92 % both at full-load and half-load at unity power factor. Determine its efficiency at 60 % of full-load at 0.8 power factor lag.

Solution:

$$\eta = \frac{x \times \text{kVA} \times \cos\varphi}{(x \times \text{kVA}) \times \cos\varphi + x^2 \times w_{cu} + w_i} \times 100$$

where x represents percentage of full-load, W_i is iron loss and W_{Cu} is full-load Cu loss.

At F.L. u.p.f. Here x = 1

$$\therefore\ 92 = \frac{1 \times 600 \times 1}{(1 \times 600) \times 1 + 1^2 \times w_{cu} + w_i} \times 100$$

∴ $W_i + W_{Cu}$ = 52.174 kW ...**(i)**

At half F.L. UPF. Here x = 1/2

$$92 = \frac{\frac{1}{2} \times 600 \times 1}{\left(\frac{1}{2} \times 600\right) \times 1 + \left(\frac{1}{2}\right)^2 \times w_{cu} + w_i} \times 100$$

W_i + 0.25 W_{Cu} = 26.087 kW ...**(ii)**

From **(i)** and **(ii)**, we get, W_i = 17.39 kW, W_{Cu} = 34.78 kW

60 % F.L. 0.8 p.f. (lag) Here, x = 0.6

$$\eta = \frac{0.6 \times 600 \times 0.8}{(0.6 \times 600) \times 0.8 + 0.6^2 \times 34.78 + 17.39} \times 100 = \mathbf{85.9\ \%}$$

Example 65: A 600-kVA, 1-ph transformer when working at u.p.f. has an efficiency of 92 % at full-load and also at half-load. Determine its efficiency when it operates at unity p.f. and 60 % of full-load.

Solution:

The fact that efficiency is the same i.e. 92 % at both full-load and half-load will help us to find the iron and copper losses.

At full-load

Output = 600 kW ; Input = 600/0.92 = 652.2 kW ; Total loss = 652.2 − 600 = 52.2 kW

Let x = Iron loss − It remains constant at all loads.

y = F.L. Cu loss − It is $\propto$ $(kVA)^2$. $\therefore$ x + y = 52.2 ...**(i)**

At half-load

Output = 300 kW ; Input = 300/0.92 $\therefore$ Losses = (300/0.92 − 300) = 26.1 kW

Since Cu loss becomes one-fourth of its F.L. value, hence

x + y/4 = 26.1

Solving for x and y, we get x = **17.4 kW** ; y = 34.8 kW ...**(ii)**

At 60 % full-load

Cu loss = $0.6^2 \times 34.8 = 12.53$ kW ; Total loss = 17.4 + 12.53 = 29.93 kW

Output = 600 × 0.6 = 360 kW $\therefore$ η = 360/389.93 = 0.965 or **96.5 %**

Example 66: The maximum efficiency of a 100-kVA, single phase transformer is 98% and occurs at 80% of full load at 8 p.f. If the leakage impedance of the transformer is 5 %, find the voltage regulation at rated load of 0.8 power factor lagging.

Solution:

Since maximum efficiency occurs at 80 percent of full-load at 0.8 p.f.,

Output at η_{max} = (100 × 0.8) × 0.8 = 64 kW ;

Input = 64/0.98 = 65.3 kW

$\therefore$ Total loss = 65.3 − 64 = 1.3 kW. This loss is divided equally between Cu and iron.

$\therefore$ Cu loss at 80 % of full-load = 1.3/2 = 0.65 kW

Cu loss at full-load = $0.65/0.8^2$ = 1 kW

$$\% R = \frac{\text{Cu loss}}{V_2 I_2} \times 100 = 1 \times \frac{100}{100} = 1\% = v_r \; ; \qquad v_x = 5\ \%$$

$$\therefore \ \%\text{ age regn.} = (1 \times 0.8 + 5 \times 0.6) + \frac{1}{200} \times (5 \times 0.8 - 1 \times 0.6)^2 = \mathbf{0.166\ \%} \quad \text{(exact)}$$

Example 67: A 10 kVA, 5000/440-V, 25-Hz single phase transformer has copper, eddy current and hysteresis losses of 1.5, 0.5 and 0.6 per cent of output on full load. What will be the percentage losses if the transformer is used on a 10-kV, 50-Hz system keeping the full-load current constant ? Assume unity power factor operation. Compare the full load efficiencies for the two cases.

Solution:

We know that E_1 = 4,44 f $N_1 B_1 A$.. When both excitation voltage and frequency are doubled, flux remains unchanged.

F.L. output at upf = 10 kVA × 1 = 10 kW

F.L. Cu loss = 1.5 × 10/100 = 0.15 kW ;

Eddy current loss (W_e) = 0.5 × 10/100 = 0.05 kW

Hysteresis loss (W_h) = 0.6 × 10/100 = 0.06 kW

Now, full-load current is kept constant but voltage is increased from 5000 V to 10,000 V. Hence, output will be doubled to 20 kW. Due to constant current, Cu loss would also remain constant.

New Cu loss = 0.15 kW, % Cu loss = (0.15/20) × 100 = 0.75 %

Now, $W_e \propto f^2$ and $W_h \propto f$.

New W_e = 0.05 $(50/25)^2$ = 0.2 kW, % W_e = (0.2/20) × 100 = 1 %

Now, W_h = 0.06 × (50/25) = 0.12 kW, % W_h = (0.12/20) × 100 = 0.6 %

$$\eta_1 = \frac{10}{10+0.15+0.05+0.06} \times 100 = \mathbf{87.4\ \%}$$

$$\eta_2 = \frac{20}{20+0.15+0.2+0.12} \times 100 = \mathbf{97.7\ \%}$$

Example 68: A 300-kVA, single-phase transformer is designed to have a resistance of 1.5 % and maximum efficiency occurs at a load of 173.2 kVA. Find its efficiency when supplying full-load at 0.8 p.f. lagging at normal voltage and frequency.

Solution:

$$\% R = \frac{\text{F.L.Cu loss}}{\text{F.L.}V_2 I_2} \times 100$$

$$1.5 = \frac{\text{F.L.Cu loss}}{300 \times 1000} \times 100$$

∴ F.L. Cu loss = 1.5 × 300 × 1000/100 = 4500 W

$$\text{Also, } 173.2 = 300\sqrt{\left(\frac{\text{Iron loss}}{4500}\right)} \quad ;\ \text{Iron loss} = 1500\ \text{W}$$

Total F.L. loss = 4500 + 1500 = 6 kW

$$\text{F.L. } \eta \text{ at 0.8 p.f.} = \frac{300 \times 0.8}{(300 \times 0.8) + 6} \times 100 = \mathbf{97.6\ \%}$$

Example 69: A single phase transformer is rated at 100-kVA, 2300/230-V, 50 Hz. The maximum flux density in the core is 1.2 Wb/m^2 and the net cross-sectional area of the core is 0.04 m^2. Determine

(a) The number of primary and secondary turns needed.

(b) If the mean length of the magnetic circuit is 2.5 m and the relative permeability is 1200, determine the magnetising current. Neglect the current drawn for the core loss.

(c) On short-circuit with full-load current flowing, the power input is 1200 W and an open circuit with rated voltage, the power input was 400 W. Determine the efficiency of the transformer at 75 % of full-load with 0.8 p.f. lag.

(d) If the same transformer is connected to a supply of similar voltage but double the frequency (i.e., 100 Hz). What is the effect on its efficiency ?

Solution:

(a) Applying e.m.f. equation of the transformer to the primary, we have

$2300 = 4.44 \times 50 \times N_1 \times (1.2 \times 0.0.4)$

$\therefore\ N_1 =$ **216**

$K = 230/2300 = 1/10$

$N_2 = KN_1 = 216/10 = 21.6$ or **22**

(b) $AT = H \times \boldsymbol{l} = \dfrac{B}{\mu_0 \mu_r} \times \boldsymbol{l} = \dfrac{1.2}{4\pi \times 10^{-7} \times 1200} \times 2.5 = 1989$

$\therefore$ I =1989 / 216 = **9.21 A**

(c) F.L. Cu loss = 1200 W − S.C. test

Iron loss = 400 W − O.C. test

Cu loss at 75 % of F.L. = $(0.75)^2 \times 1200 = 675$ W

Total loss = 400 + 675 = 1075 kW

Output = 100 × (3/4) × 0.8 = 60 kW ;

$\eta = (60/61.075) \times 100 =$ **98.26 %**

(d) When frequency is doubled, iron loss is increased because

(i) hysteresis loss is doubled $W_h \propto f$

(ii) eddy current loss is quadrupled $W_e \propto f^{\,2}$

Hence, efficiency will be decreased.

Example 70: A transformer has a resistance of 1.8 % and a reactance of 5.4 %. (a) At full load, what is the power-factor at which the regulation will be : (i) Zero, (ii) positive-maximum ? (b) If its maximum efficiency occurs at full-load (at unity p.f.), what will be the efficiency under these conditions ?

Solution:

Approx. percentage regulation is given, in this case, by the $1.8 \cos\varphi \pm 5.4 \sin\varphi$.

(a) Regulation :

(i) If regulation is zero, negative sign must be applicable. This happens at leadings p.f.
Corresponding p.f. = tan φ = 1.8/5.4 = 0.333 leading, φ = **18.440 leading**

(ii) For maximum positive regulation, lagging p.f. is a must. From phasor diagram, the result can be obtained.
Corresponding tan φ = 5.4/1.8 = 3, φ = 71.56 lagging
% Voltage regulation = 1.8 cos φ + 5.4 sin φ = **5.7 %**

(b) Efficiency : Maximum efficiency occurs at such a load when
Iron losses = Copper losses, This means Iron-losses are 1.8 %.
Efficiency = 100/(100 + 1.8 + 1.8) = **96.52 %**

Example 71: A 10 kVA, 1 phase, 50 Hz, 500/250 V transformer gave following test results :
OC test (LV) side : 250 V, 3.0 A, 200 W
SC test (LV) side : 15 V, 30 A, 300 W.
Calculate efficiency and regulation at full load, 0.8 p.f. lagging.

Solution:

For efficiency calculations, full load current should be calculated, on the L.V. side in this case,
F.L. Current =10, 000 / 250 = 40 amp
Short-circuit test data have been given at 30 A current on the L.V. side.
I^2R losses at 40 A L.V. side = $(40 / 30)^2 \times 300$ = 533.3 Watts
At rated voltage, iron losses (from O.C. test) = 200 Watts
F.L. Output at 0.8 P.F. = 10,000 × 0.8 = 8000 Watts

Hence, $\eta = \frac{8000}{8000+4733.3} \times 100\ \%$ = **91.6 %**

For regulation, series resistance and reactance parameters of the equivalent circuit have to be evaluated, from the S.C. test.
Series Impedance, Z = 15 / 30 = 0.5 Ω, Series resistance, R = 300 / (30 × 30) = 0.333 Ω
Series reactance, $x = \sqrt{0.5^2 - 0.333^2}$ = **0.373 Ω**
By Approximate formula,
p.u. regulation at full load, 0.8 p.f. lag = $\frac{40}{250} \times [0.333\times0.8 - 0.373\times0.6] = 6.82 \times 10^{-3}$ p.u.
When converted into volts, this is $6.82 \times 10^{-3} \times 250$ = **1.70 volt**

Example 72: A 40 kVA, 1-ph, transformer has an iron loss of 400 W, and full copper loss of 800 W. Find the load at which maximum efficiency is achieved at unity power factor.

Solution:

If x = fraction of rated load at which the efficiency is maximum.

P_i = Iron loss = 400 W

P_c = F.L. copper loss = 800 W

Then $x^2 P_c = P_i$

On substitution of numerical values of P_i and P_c, we get x = 0.707

Hence, the efficiency is maximum, at unity p.f. and at 70.7 % of the rated Load. At this load,

Copper loss = Iron loss = 0.40 kW

Corresponding output = 40 × 0.707 × 1 = 28.28 kW

Corresponding efficiency = 28.28 / (28.28 + 0.4 + 0.4) = **97.25 %**

Extension to Question : (a) At what load (s) at unity p.f. the efficiency will be 96.8 % ?

Solution. Let x = Fractional load at which the concerned efficiency occurs, at unity p.f.

$$\frac{40x}{40x+0.8x^2+0.4} = 0.968$$

This gives the following values of x : $x_1 = 1.25$ $x_2 = 0.40$

Thus, at 40 % and at 125 % of the rated load, the efficiency will be **96.8 %** as marked on the graph, in Figure (46).

(b) How will maximum-efficiency condition be affected if the power factor is 0.90 lagging ?

The condition for efficiency-variation-statement is that the power factor remains constant. Thus, for 0.90 lagging p.f., another curve (Lower curve in Figure (46)) will be drawn for which the maximum efficiency will occur at the same value of x (= 0.707), but

$$\eta_{max} = \frac{40x\cos\varphi}{40x\cos\varphi+0.8x^2+0.4} = \mathbf{97\ \%}$$

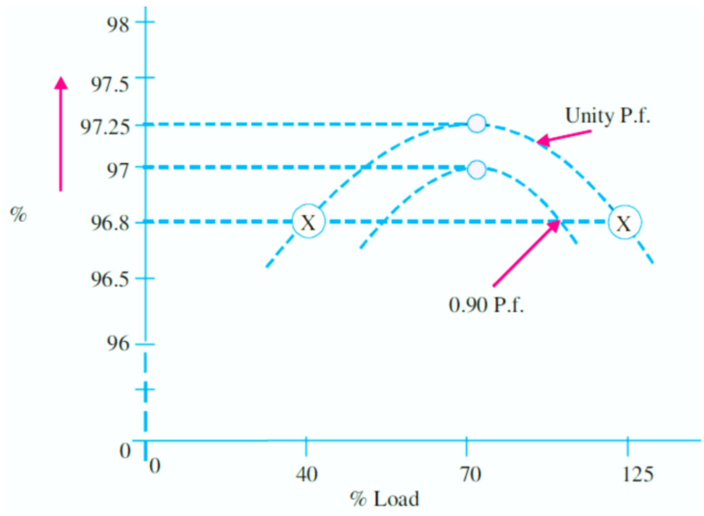


Figure (46)

Example 73: A 10 kVA, 500/250 V, single-phase transformer gave the following test results:
S.C. Test (H.V. side) : 60 V, 20 A, 150 W
The maximum efficiency occurs at unity power factor and at 1.20 times full-load current. Determine full-load efficiency at 0.80 p.f. Also calculate the maximum efficiency.

Solution:
Full-load current on H.V. side = 10,000/500 = 20 Amp
S.C. test has been conducted from H.V. side only. Hence, full-load copper-loss, at unity p.f. = 150 watts
(a) Maximum efficiency occurs at 1.2 times full-load current, at unity p.f. corresponding copper loss = $(1.2)^2 \times 150 = 216$ watts
At maximum efficiency, copper-loss = core-loss = 216 watts
Corresponding Power-output = 1.2 × 10,000 × 1.0 = 12 kW
Hence, maximum efficiency at unity P.f. = (12)/(12 + 0.216 + 0.2160) = 0.9653 = 96.53 %
(b) Full-load efficiency at 0.80 P.f.
Output Power at full-load, 0.80 P.f. = 10,000 × 0.8 = 8000 W,
constant core-loss = 216 W
Corresponding copper-loss = 150 W
Total losses = 366 W
Hence, efficiency = (8000/8366) × 100 % = 95.63 %.

23. Variation of Efficiency with Power Factor

The efficiency of a transformer is given by

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input+Losses}}{\text{Input}} = 1 - \frac{\text{Losses}}{\text{Input}} = 1 - \frac{\text{Losses}}{(V_2 I_2 \cos\varphi + \text{Losses})}$$

Let, losses/$V_2\,I_2 = x$

$$\therefore \quad \eta = 1 - \frac{\text{Losses}/V_2I_2}{\cos\varphi + (\text{Losses}/V_2I_2)}$$

$$= 1 - \frac{x}{\cos\varphi + x} = 1 - \frac{x/\cos\varphi}{1+(x/\cos\varphi)}$$

The variations of efficiency with power factor at different loadings on a typical transformer are shown in Figure (47).

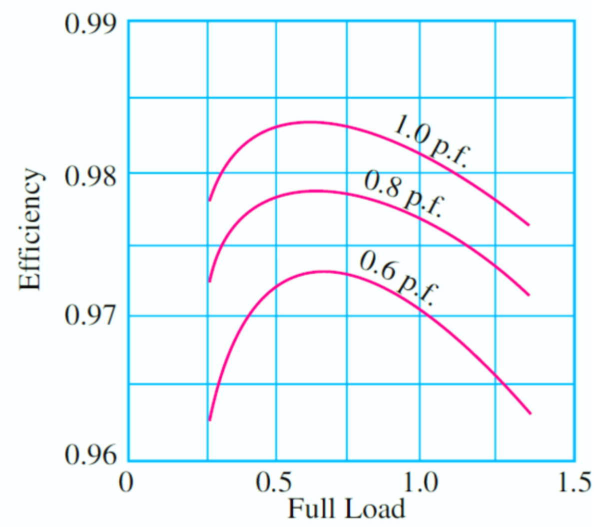


Figure (47)

Tutorial Problems (4)

[1] A 200-kVA transformer has an efficiency of 98 % at full-load. If the maximum efficiency occurs at three-quarters of full-load, calculate (a) iron loss at F.L. (b) Cu loss at F.L. (c) efficiency at half-load. Ignore magnetising current and assume a p.f. of 0.8 at all loads. **[(a) 1.777 kW (b) 2.09 kW (c) 97.92%]**

[2] A 600 kVA, 1-ph transformer has an efficiency of 92 % both at F.L. and half-load at u.p.f. Determine its efficiency at 60 % of full load at 0.8 power factor lag. **[90.59%]**

[3] Find the efficiency of a 150 kVA transformer at 25 % full load at 0.8 p.f. lag if the copper loss at full load is 1600 W and the iron loss is 1400 W. Ignore the effects of temperature rise and magnetizing current. **[96.15%]**

[4] The F.L. Cu loss and iron loss of a transformer are 920 W and 430 W respectively. (i) Calculate the loading of the transformer at which efficiency is maximum (ii) what would be the losses for giving maximum efficiency at 0.85 of full-load if total full-load losses are to remain unchanged ? **[(a) 68.4% of F.L. (ii) W_i = 565 W ; W_{cu} = 785 W]**

[5] At full-load, the Cu and iron losses in a 100-kVA transformer are each equal to 2.5 kW. Find the efficiency at a load of 65 kVA, power factor 0.8. **[93.58%]**

[6] A transformer, when tested on full-load, is found to have Cu loss 1.8% and reactance drop 3.8%. Calculate its full-load regulation (i) at unity p.f. (ii) 0.8 p.f. lagging (iii) 0.8 p.f. leading. **[(i) 1.80% (ii) 3.7 % (iii) −0.88%]**

[7] With the help of a vector diagram, explain the significance of the following quantities in the open circuit and short-circuit tests of a transformer (a) power consumed (b) input voltage (c) input current.
When a 100-kVA single-phase transformer was tested in this way, the following data were obtained : On open circuit, the power consumed was 1300 W and on short-circuit the power consumed was 1200 W. Calculate the efficiency of the transformer on (a) full-load (b) half-load when working at unity power factor. **[(a) 97.6% (b) 96.9%]**

[8] An 11,000/230-V, 150-kVA, 50-Hz, 1-phase transformer has a core loss of 1.4 kW and full-load Cu loss of 1.6 kW. Determine (a) the kVA load for maximum efficiency and the minimum efficiency (b) the efficiency at half full-load at 0.8 power factor lagging. **[140.33 kVA, 97.6% ; 97%]**

[9] A 1-phase transformer, working at unity power factor has an efficiency of 90 % at both half-load and a full-load of 500 kW. Determine the efficiency at 75 % of F.L. **[90.5%]**

[10] A 10-kVA, 500/250-V, single-phase transformer has its maximum efficiency of 94 % when delivering 90 % of its rated output at unity power factor. Estimate its efficiency when delivering its full-load output at p.f. of 0.8 lagging. **[92.6%]**

[11] A single-phase transformer has a voltage ratio on open-circuit of 3300/660-V. The primary and secondary resistances are 0.8 Ω and 0.03 Ω respectively, the corresponding leakage reactance being 4 Ω and 0.12 Ω. The load is equivalent to a coil of resistance 4.8 Ω and inductive reactance 3.6 Ω. Determine the terminal voltage of the transformer and the output in kW. **[636 V, 54 kW]**

[12] A 100-kVA, single-phase transformer has an iron loss of 600 W and a copper loss of 1.5 kW at full load current. Calculate the efficiency at (a) 100 kVA output at 0.8 p.f. lagging (b) 50 kVA output at unity power factor. **[(a) 97.44% (b) 98.09%]**

[13] A 10-kVA, 440/3300-V, 1-phase transformer, when tested on open circuit, gave the following figures on the primary side : 440V ; 1.3 A ; 115 W.
When tested on short-circuit with full-load current flowing, the power input was 140 W. Calculate the efficiency of the transformer at (a) full-load unity p.f. (b) one quarter full-load 0.8 p.f. **[(a) 97.51% (b) 94.18%]**

[14] A 150-kVA single-phase transformer has a core loss of 1.5 kW and a full-load Cu loss of 2 kW. Calculate the efficiency of the transformer (a) at full-load, 0.8 p.f. lagging (b) at one-half full-load unity p.f. Determine also the secondary current at which the efficiency is maximum if the secondary voltage is maintained at its rated value of 240 V.
[(a) 97.17% (b) 97.4% ; 541 A]

[15] A 200-kVA, 1-phase, 3300/400-V transformer gave the following results in the short-circuit test. With V applied to the primary and the secondary short-circuited, the primary current was the full-load value and the input power 1650 W. Calculate the secondary p.d. and percentage regulation when the secondary load is passing 300 A at 0.707 p.f. lagging with normal primary voltage. **[380 V ; 480%]**

[16] The primary and secondary windings of a 40-kVA, 6600/250-V, single-phase transformer have resistances of 10 Ω and 0.02 Ω respectively. The leakage reactance of the transformer referred to the primary is 35 Ω. Calculate (a) the primary voltage required to circulate full-load current when the secondary is short-circuited. (b) the full-load regulations at (i) unity (ii) 0.8 lagging p.f. Neglect the no-load current.
[(a) 256 V (b) (i) 2.2% (ii) 3.7%]

[17] Calculate :
(a) F.L. efficiency at unity p.f.
(b) The secondary terminal voltage when supplying full-load secondary current at p.f. (i) 0.8 lag
(ii) 0.8 lead for the 4-kVA, 200/400 V, 50 Hz, 1-phase transformer of which the following are the test figures :
Open circuit with 200 V supplied to the primary winding-power 60 W. Short-circuit with 16 V applied to the h.v. winding-current 8 A, power 40 W. **[0.97 ; 383 V ; 406 V]**

[18] A 100-kVA, 6600/250-V, 50-Hz transformer gave the following results :
O.C. test : 900 W, normal voltage.
S.C. test (data on h.v. side) : 12 A, 290 V, 860 W
Calculate
(a) the efficiency and percentage regulation at full-load at 0.8 p.f. lagging.
(b) the load at which maximum efficiency occurs and the value of this efficiency at p.f. of unity, 0.8 lag and 0.8 lead. **[(a) 97.3%, 4.32% (b) 81 kVA, 97.8%, 97.3% ; 97.3%]**

[19] The primary resistance of a 440/110-V transformer is 0.5 Ω and the secondary resistance is 0.04 Ω.
When 440 V is applied to the primary and secondary is left open-circuited, 200 W is drawn from the supply. Find the secondary current which will give maximum efficiency and calculate this efficiency for a load having unity power factor. **[53 A ; 93.58%]**

[20] Two tests were performed on a 40-kVA transformer to predetermine its efficiency. The results were:
Open circuit : 250 V at 500 W
Short circuit : 40 V at F.L. current, 750 W both tests from primary side. Calculate the efficiency at rated kVA and 1/2 rated kVA at (i) unity p.f. (ii) 0.8 p.f. **[96.97% ; 96.68% ; 96.24% ; 95.87%]**

[21] The following figures were obtained from tests on a 30-kVA, 3000/110-V transformer :
O.C. test : 3000 V 0.5 A 350 W
S.C. test ; 150 V 10 A 500 W
Calculate the efficiency of the transformer at (a) F.L., 0.8 p.f. (b) half-load, u.p.f. Also, calculate the kVA output at which the efficiency is max. **[96.56% ; 97% ; 25.1 kVA]**

[22] The efficiency of a 400 kVA, 1-phase transformer is 98.77 % when delivering full load at 0.8 power factor, and 99.13 % at half load and unity power factor. Calculate (a) the iron loss, (b) the full load copper loss. **[(a) 1012 W (b) 2973 W]**

24. All-day Efficiency

The ordinary or commercial efficiency of a transformer is given by the ratio

$$\frac{\text{Output in watts}}{\text{Input in watts}}$$

But there are certain types of transformers whose performance cannot be judged by this efficiency. Transformers used for supplying lighting and general network i.e., distribution transformers have their primaries energised all the twenty-four hours, although their secondaries supply little or no-load much of the time during the day except during the house lighting period. It means that whereas core loss occurs throughout the day, the Cu loss occurs only when the transformers are loaded. Hence, it is considered a good practice to design such transformers so that core losses are very low. The Cu losses are relatively less important, because they depend on the load. The performance of such is compared on the basis of energy consumed during a certain time period, usually a day of 24 hours.

$$\therefore \eta_{\text{all-day}} = \frac{\text{Output in kWh}}{\text{Input in kWh}} \quad \text{(For 24 hours)}$$

This efficiency is always less than the commercial efficiency of a transformer.

To find this all-day efficiency or (as it is also called) energy efficiency, we have to know the load cycle on the transformer i.e., how much and how long the transformer is loaded during 24 hours. Practical calculations are facilitated by making use of a **load factor**.

Example 74: Find the all-day efficiency of 500-kVA distribution transformer whose copper loss and iron loss at full load are 4.5 kW and 3.5 kW respectively. During a day of 24 hours, it is loaded as under :

No. of hours	Loading in kW	Power factor
6	400	0.8
10	300	0.75
4	100	0.8
4	0	–

Solution:

It should be noted that a load of 400 kW at 0.8 p.f. is equal to 400/0.8 = 500 kVA.

Similarly, 300 kW at 0.75 p.f. means 300/0.75 = 400 kVA and 100 kW at 0.8 p.f. means 100/0.8 = 125 kVA

i.e., one-fourth of the full-load.

Cu loss at F.L. of 500 kVA = 4.5 kW

Cu loss at 400 kVA = $4.5 \times (400/500)^2 = 2.88$ kW

Cu loss at 125 kVA = $4.5 \times (125/500)^2 = 0.281$ kW

Total Cu loss in 24 hrs = $(6 \times 4.5) + (10 \times 2.88) + (4 \times 0.281) + (4 \times 0) = 56.924$ kWh

The iron loss takes place throughout the day irrespective of the load on the transformer because its primary is energized all the 24 hours.

The world first 5,000 KVA amorphous transformer

Iron loss in 24 hours = 24 × 3.5 = 84 kWh

Total transformer loss = 56.924 + 84 = 140.924 kWh

Transformer output is 24 hrs = (6 × 400) + (10 × 300) + (4 × 100) = 5800 kWh

$\eta_{all\text{-}day}$ = output / (output + losses) = 5800 / (5800 + 140.924) = 0.976 or **97.6 %**

Example 75: A 100-kVA lighting transformer has a full-load loss of 3 kW, the losses being equally divided between iron and copper. During a day, the transformer operates on full-load for 3 hours, one half-load for 4 hours, the output being negligible for the remainder of the day. Calculate the all-day efficiency.

Solution:

It should be noted that lighting transformers are taken to have a load p.f. of unity.

Iron loss for 24 hour = 1.5 × 24 = 36 kWh ; F.L. Cu loss = 1.5 kW

Cu loss for 3 hours on F.L. = 1.5 × 3 = 4.5 kWh

Cu loss at half full-load = 1.5/4 kW

Cu loss for 4 hours at half the load = (1.5/4) × 4 = 1.5 kWh

Total losses = 36 + 4.5 + 1.5 = 42 kWh

Total output = (100 × 3) + (50 × 4) = 500 kWh

$\eta_{all\text{-}day}$ = 500 × 100/542 = **92.26 %**

Incidentally, ordinary or commercial efficiency of the transformer is = 100/(100 + 3)
= 0.971 or **97.1 %**

Example 76: Two 100-kW transformers each has a maximum efficiency of 98 % but in one the maximum efficiency occurs at full-load while in the other, it occurs at half-load. Each transformer is on full-load for 4 hours, on half-load for 6 hours and on one-tenth load for 14 hours per day. Determine the all-day efficiency of each transformer.

Solution:

Let x be the iron loss and y the full-load Cu loss. If the ordinary efficiency is a maximum at (1/m) of full-load, then $x = y/m^2$.

Now, output = 100 kW ; Input = 100/0.98

Total losses = 100/0.98 − 100 = 2.04 kW

$y + (x/m^2) = 2.04$

1st Transformer

Here m = 1 ; y + y = 2.04 ; y = 1.02 kW and x = 1.02 kW

Iron loss for 24 hours = 1.02 × 24 = 24.48 kWh

Cu loss for 24 hours = $4 \times 1.02 + 6 \times (1.02/4) + 14\,(1.02/10^2) = 5.73$ kWh

Total loss = 24.48 + 5.73 = 30.21 kWh = 4 × 100 + 6 × 50 + 14 × 10 = 840 kWh

$\eta_{all\text{-}day}$ = 840/870.21 = 0.965 or **96.5 %**

2nd Transformer

Here 1/m = 1/2 or m = 2 ∴ y + (y/4) = 2.04

or y = 1.63 kW ; x = 0.14 kW

Output = 840 kWh ...as above

Iron loss for 24 hours = 0.41 × 24 = 9.84 kWh

Cu loss for 24 hours = 4 × 1.63 + 6 (1.63/4) + 14 ($1.63/10^2$) = 9.19 kWh

Total loss = 9.84 + 9.19 = 19.03 kWh

$\eta_{all\text{-}day}$ = 840/859.03 = 0.978 or **97.8 %**

Example 77: A 5-kVA distribution transformer has a full-load efficiency at unity p.f. of 95%, the copper and iron losses then being equal. Calculate its all-day efficiency if it is loaded throughout the 24 hours as follows :

No load for	10 hours	Quarter load for	7 hours
Half load for	5 hours	Full load for	2 hours

Assume load p.f. of unity.

Solution:

Let us first find out the losses from the given commercial efficiency of the transformer.

Output = 5 × 1 = 5 kW ; Input = 5/0.95 = 5.264 kW

Losses = (5.264 − 5.000) = 0.264 kW = 264W

Since efficiency is maximum, the losses are divided equally between Cu and iron.

Cu loss at F.L. of 5 kVA = 264/2 = 132 W ; Iron loss = 132 W

Cu loss at one-fourth F.L. = $(1/4)^2$ × 132 = 8.2 W

Cu loss at one-half F.L. = $(1/2)^2$ × 132 = 33 W

Quarter load Cu loss for 7 hours = 7 × 8.2 = 57.4 Wh

Half-load Cu loss for 5 hours = 5 × 33 = 165 Wh

F.L. Cu loss for 2 hours = 2 × 132 = 264 Wh

Total Cu loss during one day = 57.4 + 165 + 264 = 486.4 Wh = 0.486 kWh

Iron loss in 24 hours = 24 × 132 = 3168 Wh = 3.168 kWh

Total losses in 24 hours = 3.168 + 0.486 = 3.654 kWh

Since load p.f. is to be assumed as unity.

F.L. output = 5 × 1 = 5 kW ; Half F.L. output = (5/2) × 1 = 2.5 kW

Quarter load output = (5/4) × 1 = 1.25 kW

Transformer output in a day of 24 hours = (7 × 1.25) + (5 × 2.5) + (2 × 5) = 31.25 kWh

$$\eta_{all\text{-}day} = \frac{31.25}{31.25 + 3.654} \times 100 = \mathbf{89.53\ \%}$$

Example 78: Find "all day" efficiency of a transformer having max. efficiency of 98 % at 15 kVA at unity power factor and loaded as follows :
12 hours – 2 kW at 0.5 p.f. lag
6 hours – 12 kW at 0.8 p.f. lag
6 hours – at no load.

Solution:

Output = 15 × 1 = 15 kW, input = 15/0.98

Losses = (15/0.98 − 15) = 0.306 kW = 306 W

Since efficiency is maximum, the losses are divided equally between Cu and iron.

Cu loss at 15 kVA = 306/2 = 153 W, Iron loss = 153 W

2 kW at 0.5 p.f. = 2/0.5 = 4 kVA, 12 kW at 0.8 p.f. = 12/0.8 = 15 kVA

Cu loss at 4 kVA = 153 $(4/15)^2$ = 10.9 W ; Cu loss at 15 kVA = 153 W.

Cu loss in 12 hrs = 12 × 10.9 = 131 Wh ;

Cu loss in 6 hr = 6 × 153 = 918 Wh.

Total Cu loss for 24 hr = 131 + 918 = 1050 W_h = 1.05 kWh

Iron loss for 24 hrs = 24 × 153 = 3,672 W_h = 3.672 kWh

Output in 24 hrs = (2 × 12) + (6 × 12) = 96 kWh

Input in 24 hrs = 96 + 1.05 + 3.672 = 100.72 kWh ⇨ $\eta_{all\text{-}day}$ = 96 × 100/100.72 = **95.3 %**

Example 79: 150-kVA transformer is loaded as follows :
Load increases from zero to 100 kVA in 3 hours from 7 a.m. to 10.00 a.m., stays at 100 kVA from 10 a.m. to 6 p.m. and then the transformer is disconnected till next day. Assuming the load to be resistive and core-loss equal to full-load copper loss of 1 kW, determine the all-day efficiency and the ordinary efficiency of the transformer.

Solution:

Since load is resistive, its p.f. is unity.
Average load from 7 a.m. to 10 a.m. = (0 + 100)/2 = 50 kVA i.e., one-third F.L.
Load from 10 a.m. to 6 p.m. = 100 kVA i.e., 2/3 of F.L.

Ordinary Efficiency

In this case, load variations are not relevant.
Output = 150 × 1 = 150 kW ; Iron loss = Cu loss = 1 kW ; Total loss = 2 kW.

Ordinary η = 150/(150 + 2) = 0.9868 or **98.68 %**

All-day Efficiency

Cu loss from 7-10 a.m. = 3 × $(1/3)^2$ × 1 = 0.333 kWh
Cu loss from 10 a.m. to 6.p.m. = 8 × $(2/3)^2$ × 1 = 3.555 kWh
Total Cu loss for 24 hrs = 0.333 + 3.555 = 3.888 kWh
Total iron loss for 24 hrs = 24 × 1 = 24 kWh; Losses for a day of 24 hrs = 27.888 kWh
Output for 24 hrs = 3 × (50 × 1) + 8 (100 × 1) = 950 kWh

$$\eta_{all\text{-}day} = \frac{950}{950 + 27.888} \times 100 = \mathbf{97.15\ \%}$$

Example 80: Find the all-day efficiency of a 50 kVA distribution transformer having full load efficiency of 94 % and full-load copper losses are equal to the constant iron losses. The loading of the transformer is as follows, the power factor being 1.0.

(i) No load for 10 hours (ii) Half load for 5 hours
(iii) 25 % load for 6 hours (iv) Full load for 3 hours.

Solution:

At full load unity p.f.

efficiency = 94 % = $\frac{50000}{50000+2P_i}$

$2P_i = [\frac{50000}{0.94} - 50000]$, or $P_i = \frac{1}{2} \times 50000 [\frac{1-0.94}{0.94}]$

$\therefore$Pi = 25,000 × (0.06 / 0.94) = 1596 Watts
Hence, full load Cu-losses = 1596 Watts

(a) Energy required in overcoming Cu-losses, during 24 hours

(i) No load for 10 hours : zero

(ii) At half load, Cu-losses = $(0.5)^2$ × 1596 Watts = 399

Energy in 5 hours = $\frac{399 \times 5}{1000}$ kWh = 1.995 kWh

(iii) At 25 % load, Cu-loss = $(0.25)^2$ × 1596 = 99.75 Watts

Energy in 6 hours = $\frac{6 \times 99.75}{1000}$ = 0.5985 kWh

(iv) Energy lost during 3 hours of full load = $\frac{1596 \times 3}{1000}$ = 4.788 kWh

(b) Energy lost in constant core-losses for 2 hours = $\frac{1596}{1000}$ × 24 = 38.304 kWh

(c) Energy required by the load = 25 × 5 + 12.5 × 6 + 50 × 3 = 125 + 75 + 150 = 350 kWh

All-day efficiency = $\frac{350}{350+38.304+7.3815}$ × 100 = **88.454 %**

Example 81: A 10 kVA, 1-ph transformer has a core-loss of 40 W and full load ohmic loss of 100 W. The daily variation of load on the transformer is as follows :

6 a.m. to	1 p.m.	3 kW at 0.60 p.f.
1 p.m. to	5 p.m.	8kW at 0.8 p.f.
5 p.m. to	1 a.m.	full load at u.p.f.
1 a.m. to	6 a.m.	no load

Determine all day efficiency of the transformer

Solution:

Fractional loading (= x) and the output kWh corresponding to load variations can be worked out in tabular form, as below :

S.N.	*Number of hours*	$x = \frac{\text{load kVA}}{\text{Xmer Rating}}$	$x^2 P_c$ *in kW*	*Output in kWh*	*Copper Loss in kWh*
1	7	$\frac{3/0.6}{10} = 0.5$	$0.50^2 \times 0.10 = 0.025$	$3 \times 7 = 21$	$0.025 \times 7 = 0.175$
2	4	$\frac{8/0.8}{10} = 1.0$	0.10	$8 \times 4 = 32$	$0.1 \times 4 = 0.40$
3	8	$\frac{10/1}{10} = 1.0$	0.10	$10 \times 8 = 80$	$0.1 \times 8 = 0.8$
4	5	Zero	Zero	Zero	Zero
			Output in kWh	$21 + 32 + 80 = 133$	
			Ohmic Loss, in kWh		$0.175 + 0.40 + 0.80 = 1.375$
		Core loss during 24 Hrs = $\frac{400}{1000} \times 24 = 0.96$ kWh			

Hence, Energy efficiency (= All day Efficiency) = $\frac{133}{133+1.375+0.96} \times 100$ = **98.3 %**

Example 82: A transformer has its maximum efficiency of 0.98 at 15 kVA at unity p.f. During a day, it is loaded as follows :

12 hours :	2 kW	at	0.8 p.f.
6 hours :	12 kW	at	0.8 p.f.
6 hours	18 kW	at	0.9 p.f.

Find the all day efficiency.

Solution:

Let 15 kVA be treated as full load.

Output at maximum efficiency = 15000 × 1 watts

Input = 15000/0.98 watts

Losses = Input − Output = 15000 (1/0.98 − 1) = 15000 × 2/98 = 306 Watts

At maximum efficiency, since the variable copper-loss and constant core-loss are equal.

Full load copper-loss = Constant core-loss = 306/2 = 153 Watts

Let the term x represents the ratio of required Load/Full load.

Output = 15 × cos φ

Following tabular entries simplify the calculations for all-day efficiency.

S.N.	*x*	*Hrs*	*x^2P_c in kW*	*Energy in Copper -loss in kWh*	*Output during the period in kWh*
1	$\frac{2/0.5}{15} = \frac{4}{15}$	12	0.01088	0.131	24
2	$\frac{12/0.8}{15} = 1.0$	6	0.153	0.918	72
3	$\frac{18/0.9}{15} = 4/3$	6	0.272	1.632	108

Total output during the day = 204 kWh

Total copper-loss during the day = 2.681 kWh

Total core-loss during the day = 0.153 × 24 = 3.672

All day efficiency = (204/210.353) × 100 = 96.98 %

Tutorial Problems (5)

[1] A 100-kVA distribution transformer has a maximum efficiency of 98 % at 50 % full-load and unity power factor. Determine its iron losses and full-load copper losses.
The transformer undergoes a daily load cycle as follows :

Load	*Power factor*	*Load duration*
100 kVA	1.0	8 hrs
50 kVA	0.8	6 hrs
No load		10 hrs

Determine its all-day efficiency.

[2] What is meant by energy efficiency of a transformer ?
A 20-kVA transformer has a maximum efficiency of 98 percent when delivering three-fourth full-load at u.p.f. If during the day, the transformer is loaded as follows :

12 hours	No load
6 hours	12 kWh, 0.8 p.f.
6 hours	20 kW, u.p.f.

Calculate the energy efficiency of the transformer.